



The Golden Ratio & Fibonacci Numbers in Art, Architecture & Nature



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The Golden Ratio's Inspiration

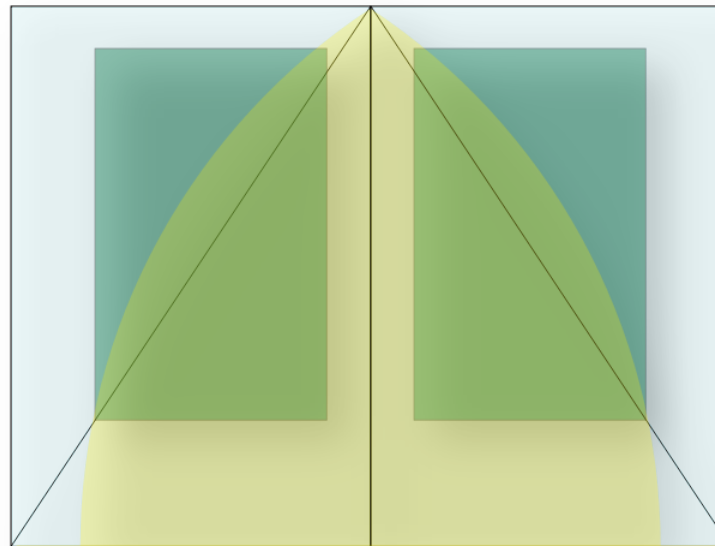
Review

What is the Golden Ratio?

- 1.618 etc.
- A proportion of beauty found in art, architecture, music nature and more mysterious places

Has inspired humanity for at least 2,400 years including:

- **Mathematicians**
- **Artists**
- **Musicians**
- **Architects**
- **Biologists**
- **Historians**
- **Psychologists**
- **Mystics**
- **Stock brokers**



Most medieval books were formatted in ideal golden proportions. The page proportion is 2:3 and the text area is proportioned in the Golden Section. Image in the public domain.



**Leonardo da Vinci's
Self Portrait**

***The Mona Lisa* and the Golden Ratio**

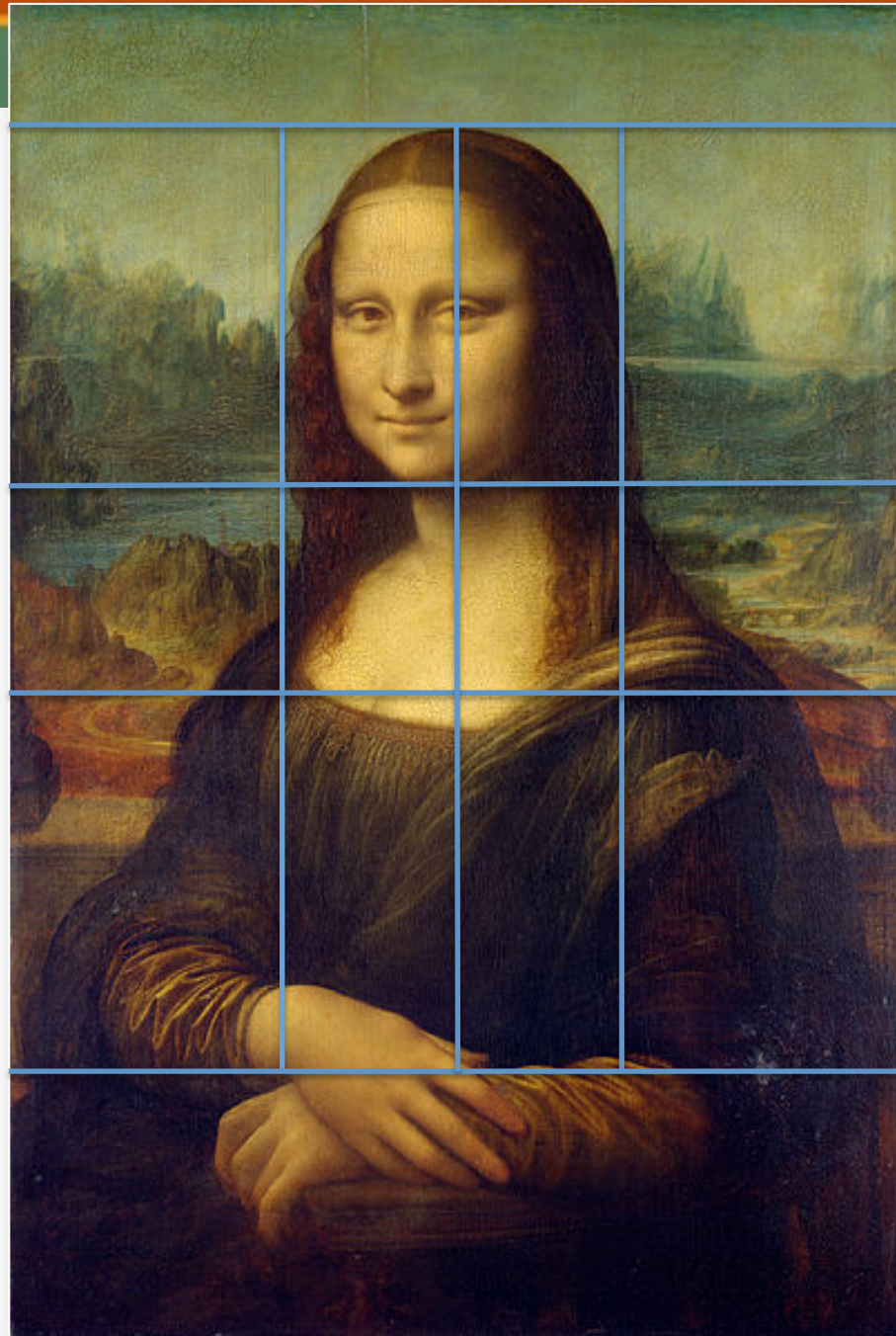
**Background Image – the
Mona Lisa showing
perspective lines.**

- "The *Mona Lisa*," – Leonardo da Vinci's most famous painting
- Believed Leonardo, as mathematician, made painting "golden" or followed golden ratio to promote mathematics in art
- Overall woman shape – fits within golden triangle with arms as base, head as triangle's tip; draw attention to face

The Mona Lisa & the Golden Ratio

Prominent Golden Ratio Elements:

- Head
- Garment neck line
- Left arm
- All reveal golden ratios



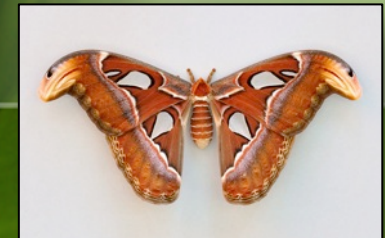
- Lines reveal Golden ratios from canvas center
- Face width close to golden ratio of canvas width
- Center of painting goes through her left eye

Fibonacci Numbers & the Golden Ratio in Nature



Nature's spirals can be made up of Fibonacci numbers in:

- Seed heads of sunflowers & daisies
- Pine cones
- Pineapples
- Nautilus shell spiral turn

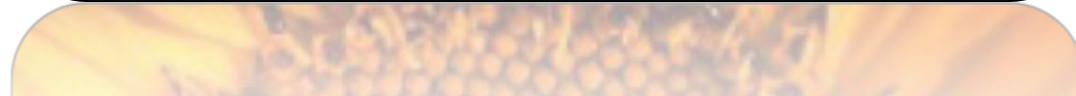
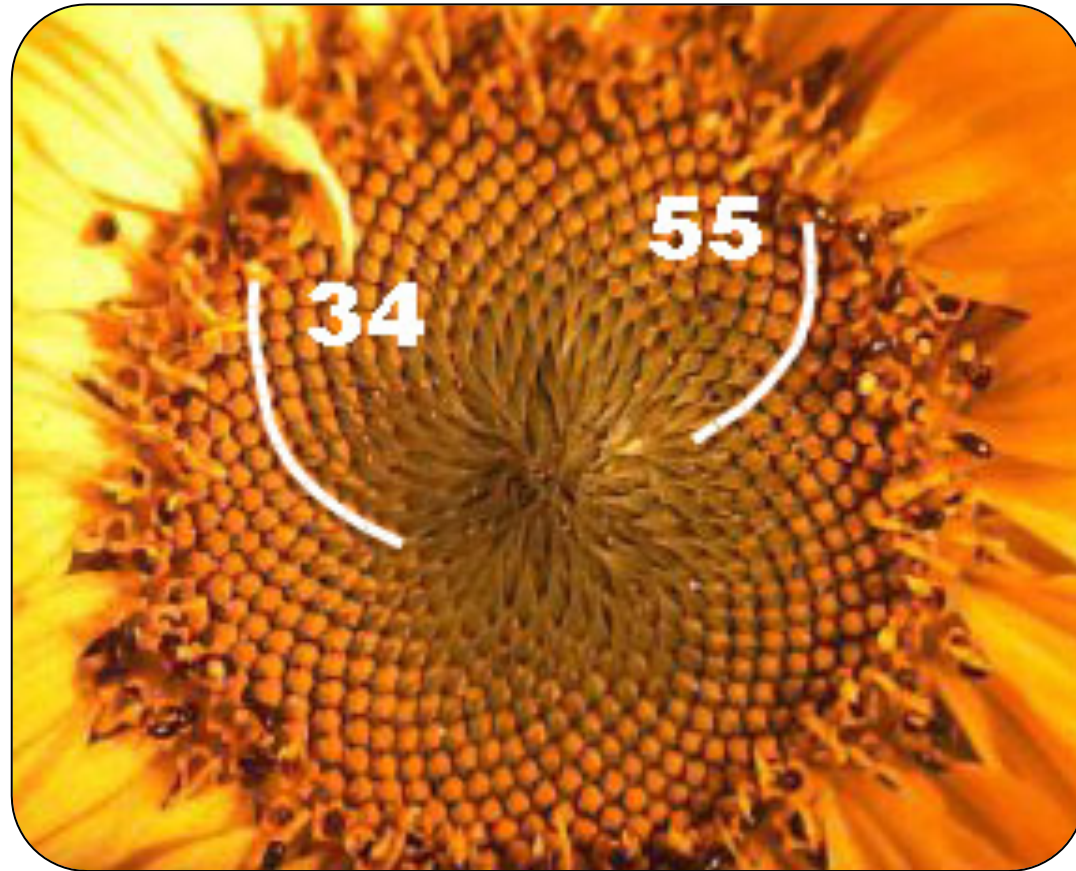


Golden rectangle can be drawn around:

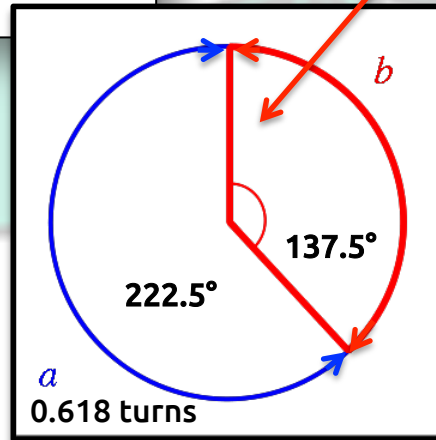
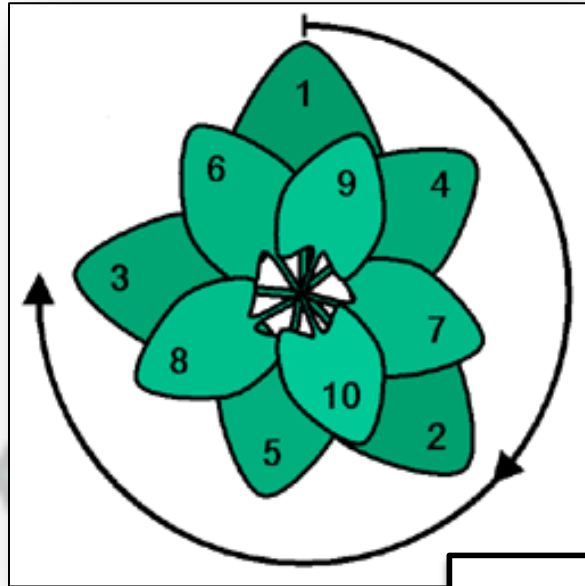
- Cecropia moths
- Nautilus shells



A sunflower revealing two Fibonacci numbers in its seed spiral turns.



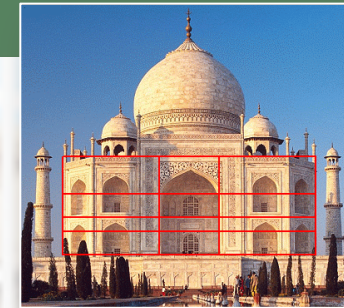
The Golden Angle & Plant Phyllotaxis



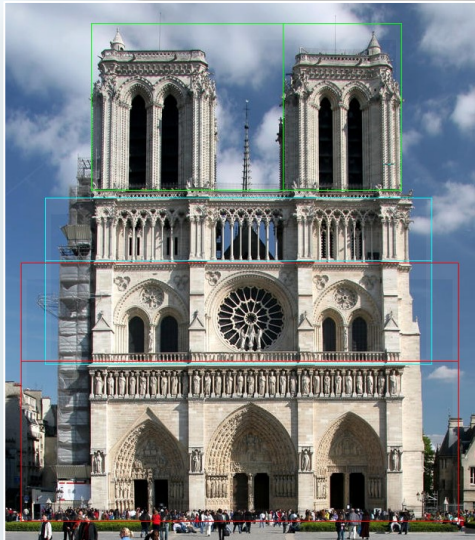
- Phyllotaxis – (Gr. leaf arrangement) in botany (plant study), leaf arrangement on an axis or stem
- Golden angle (angle between one leaf & the next) – 137.5° occupied by smaller (red) arc when two arcs making up circle are in golden ratio
- Equivalent of 0.618 rotations is 222.5° & opposite direction = 137.5°

The Golden Angle is the angle separating the florets (tiny seed head flowers) on a sunflower.

Golden Ratio in Architecture



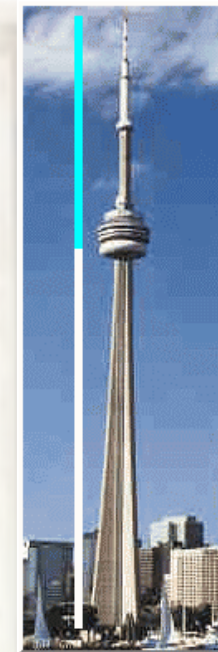
Taj Mahal constructed in the 1500's in India.



Notre Dame Cathedral in Paris, France built in 1100's. Image from [GoldenNumber.net](http://www.goldenumber.net).



United Nations Building in NYC 1947 - 1953



CN Tower in Toronto was built from 1973-1976 from [GoldenNumber.net](http://www.goldennumber.net).

Background Image – The Parthenon showing the golden section. All smaller images by Meisner, Gary B. (2013, March 5). Phi and the Golden Section in Architecture, Retrieved December 29, 2014, from <http://www.goldennumber.net>.

Golden Ratio in Art

You can find many examples of the golden ratio created by art masters. Many of these works of perfection were created by the use of golden rectangles (golden proportions or section) and golden triangles.

- Botticelli - Birth of Venus
- Leonardo Di Vinci - Mona Lisa, Vitruvian Man
- Michelangelo - Holy Family, David
- Raphael - Crucifixion
- Rembrandt - Self-Portrait
- Salvador Dali - The Sacrament of the Last Supper, The Persistence of Memory



Rembrandt self Portrait



Salvador Dali's
*"The Persistence
of Memory"*.

Background Painting – "The Last
Supper" by Leonard da Vinci

The Golden Ratio & Fibonacci Numbers In Music

Music is created with numeric value. When the golden ratio is used to create a musical piece, it becomes an example of “*living math*”. The Fibonacci number sequence is also found in music.

- 8 notes to a scale
- 3rd & 5th notes - basis of all chords
- Span (octave) of any note is 13 notes



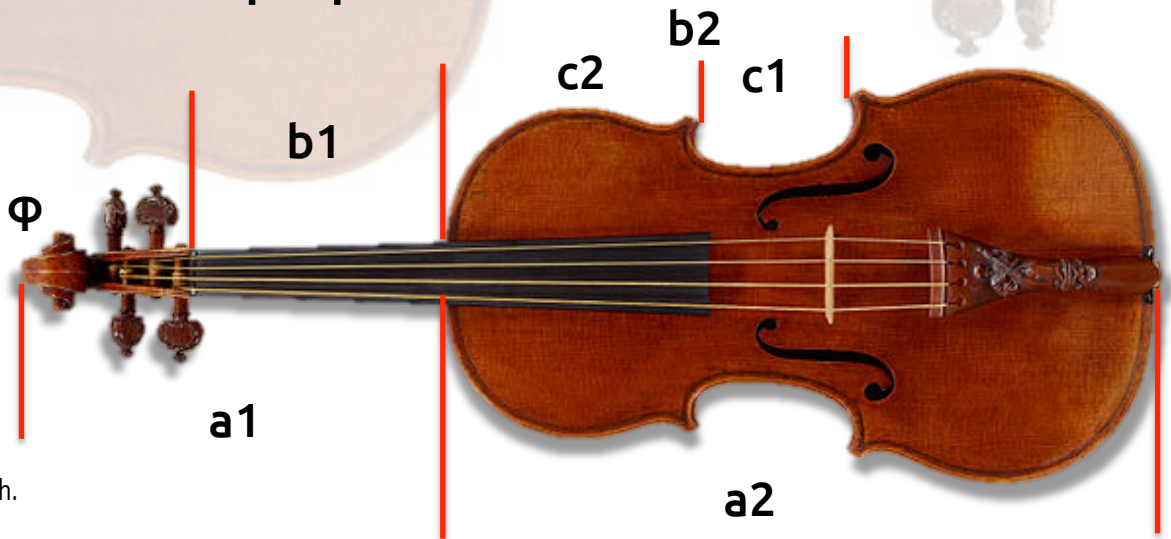
Background Image & thumbnail – A painted vase exhibiting a music lesson: teacher (right, inscription: and his student (left). Between them, a boy narrates a text, ca. 510 BC. From Vulci. Image in the public domain.

The Golden Ratio & Fibonacci & Musical Instruments



- Piano – 8 white keys, 5 black keys (grouped in 2 & 3) 13 keys total
- 3, 5, 8 and 13 belong to what number sequence?
- “Lady Blunt” (1721), famous Stradivari violin, sold for over 10 million dollars has golden ratio proportions

$$\frac{a1 + a2}{a2} = \frac{a2}{a1} = \frac{b2}{b1} = \frac{b2}{c2} = \frac{c2}{c1} = \phi$$

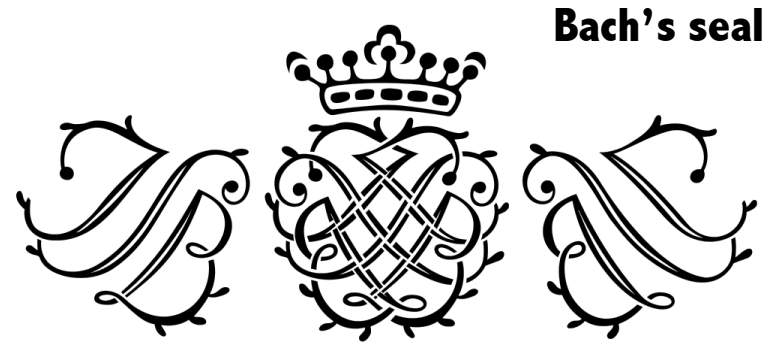


Piano keys (top image) created by Prophet of the Most High.
 Photo of the Lady Blunt violin by Tarisio Auctions

Famous Composers Use of the Golden Ratio and Fibonacci Sequence

Composers both past and present, who use(d) the golden ratio and Fibonacci sequencing in music pieces:

- Bach
- Beethoven
- Chopin
- Claude Debussy
- Liszt
- Ravel
- Schubert
- Wolfgang Mozart
- Modern composer - Casey Mongoven, uses golden ratio and Fibonacci sequence in his music



Excerpt from *fibonacci*

Patrick McCarty

Slow and steady (♩ = 60)



Music engraving by LilyPond 2.15.14—www.lilypond.org



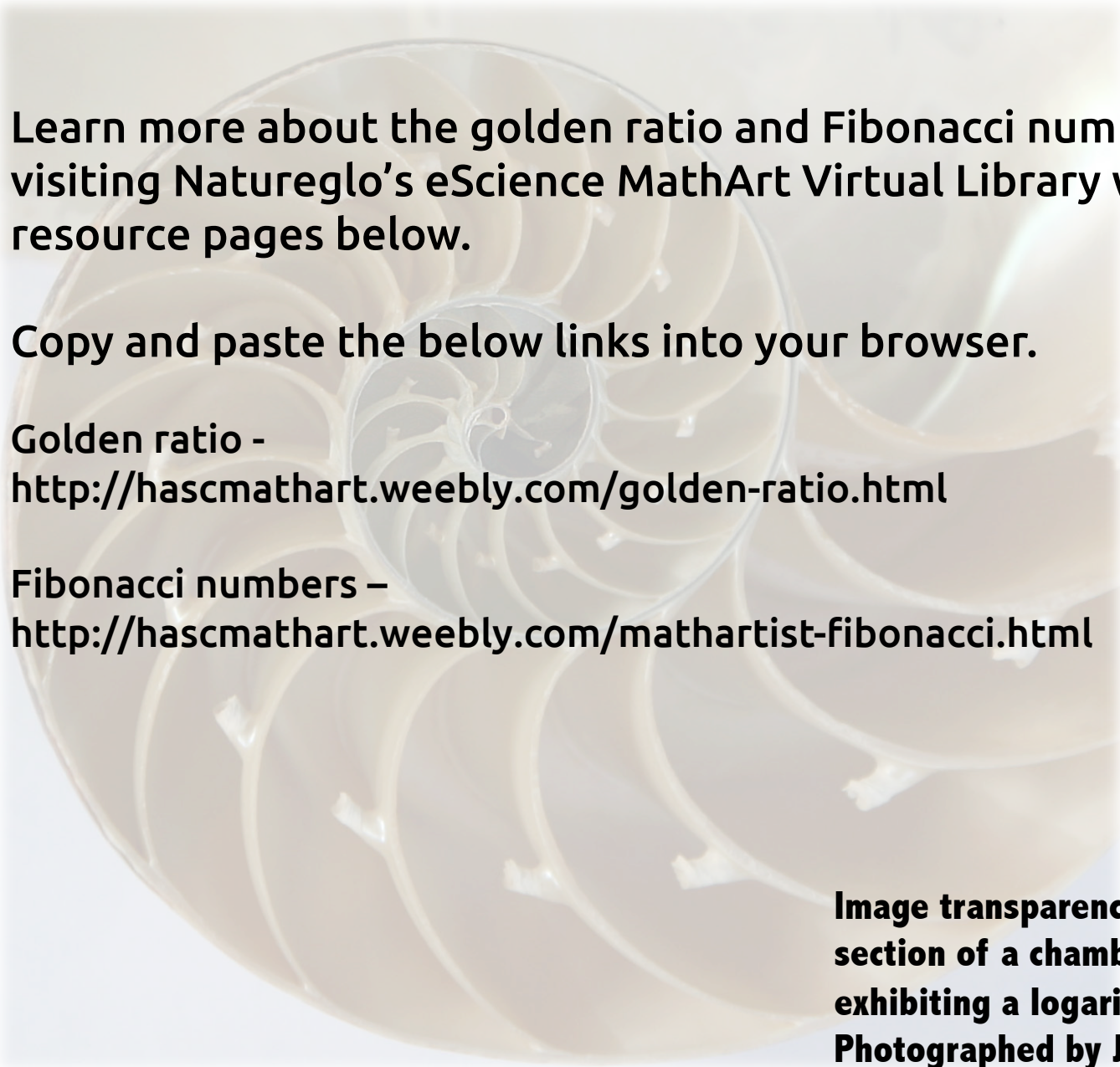
More Examples of the Golden Ratio

Phi, or the golden ratio, is found in many aspects of life and the universe. The following are several additional examples:

- Human body
- DNA spirals
- Human embryo
- Human teeth
- Penrose tiling
- Honeybees
- Tiger's face

Background Image – Human DNA computer graphics replicas.



A large, semi-transparent image of a nautilus shell cross-section is centered on the page. The shell's internal structure, including the spiral and the chambers, is clearly visible. The background of the slide is a light, neutral color.

Learn more about the golden ratio and Fibonacci numbers by visiting Natureglo's eScience MathArt Virtual Library website resource pages below.

Copy and paste the below links into your browser.

Golden ratio -

<http://hascmathart.weebly.com/golden-ratio.html>

Fibonacci numbers –

<http://hascmathart.weebly.com/mathartist-fibonacci.html>

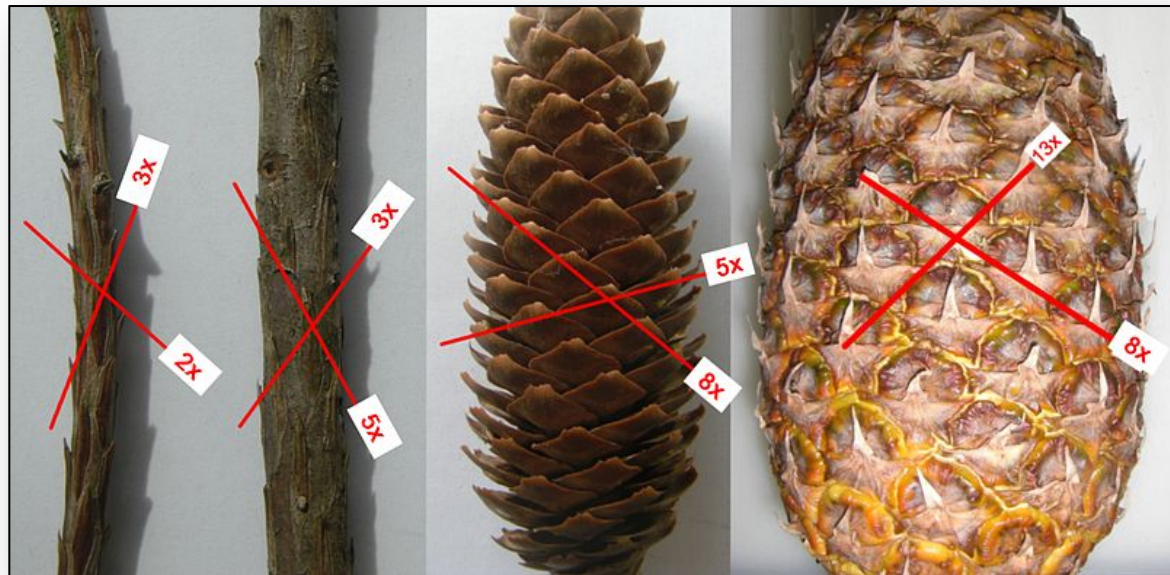
Image transparency – A cross-section of a chambered nautilus exhibiting a logarithmic spiral. Photographed by Jitze Couperus.



References Used

1. Livio, Mario (2002). *The Golden Ratio: The Story of Phi, The World's Most Astonishing Number*. New York: Broadway Books. ISBN 0-7679-0815-5.
2. Richard A Dunlap, *The Golden Ratio and Fibonacci Numbers*, World Scientific Publishing, 1997
3. Mario Livio, *The Golden Ratio: The Story of Phi, The World's Most Astonishing Number*, p.6
4. Weisstein, Eric W., "Golden Ratio", MathWorld.
5. Keith J. Devlin *The Math Instinct: Why You're A Mathematical Genius (Along With Lobsters, Birds, Cats, And Dogs)*, p. 108. New York: Thunder's Mouth Press, 2005, ISBN 1-56025-672-9
6. *Fibonacci Numbers and Nature - Part 2 : Why is the Golden section the "best" arrangement?*, from Dr. Ron Knott's *Fibonacci Numbers and the Golden Section*, retrieved 2012-11-29: <http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat2.html>

Image – The Fibonacci numbers found on both the left and right handed number of turns of two branches of *Sequoiadendron giganteum*, a cone of *Pinus strobus* and a pineapple.



A photograph of a spiral aloe plant (Aloe polyphylla) with a large, rounded base of thick, green, pointed leaves. A long, thin stem rises from the base, bearing three large, bright orange and yellow flower heads. The plant is situated in a narrow, arched passage between two walls made of dark, rough-hewn stones. The ground is dark and appears to be dirt or gravel. The text "Thank you for watching!" is overlaid in white, bold, sans-serif font at the top of the image.

Thank you for watching!

Background Image - Spiral aloe, *Aloe polyphylla*. The petals form clockwise and counter-clockwise spirals.