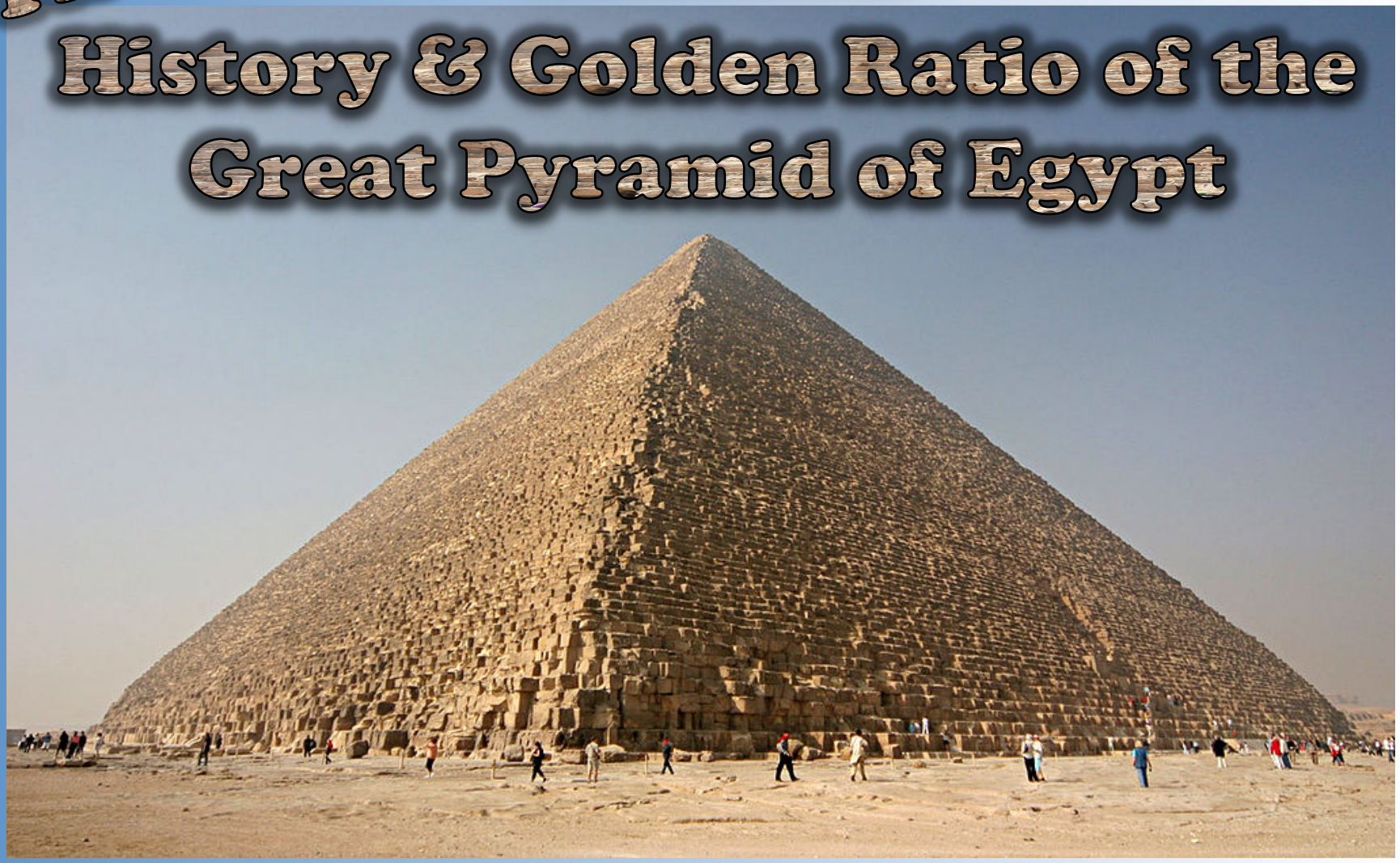
NatureGlo's eScience

The History & Golden Ratio of the Great Pyramid of Egypt



Cover image – The Great Pyramid of Egypt photographed by Nina Aldin Thune. Background image behind the pyramid are the base stones of the same pyramid photographed by Musik Animal. Text fill of the Great Pyramid's stones photographed by Bs0u10e0.

The Great Pyramid of Egypt

- Other names – Pyramid of Khufu, Pyramid of Cheops, The Great Pyramid, The Great Pyramid of Egypt, The Great Pyramid of Giza
- Oldest, largest of three Giza, Egyptian pyramids bordering what is now El Giza, Egypt

- Oldest & only one left mostly intact of all Seven Wonders of ancient world
- Believed to be built as Pharaoh Khufu's (2575-2566 BC) tomb over 20-year period
- Built 2580–2560 BC

Background image created by By Manshy482.

The Great Pyramid: One of the 7 Wonders of the Ancient World & its Builder

- Many theories of use including, a power generator and star observatory
- Original height - 146.5 meters (481 feet), tallest in world for over 3,800 years
- Son of Snefru, Khufu (second ruler of 4th dynasty) moved to royal necropolis to Giza, north of modern-day Cairo
- Khufu paid and cared for thousands of subjects to build his Great pyramid
- On Giza Plateau, builders oriented pyramid almost perfectly north

Ivory Statue of Khufu in the Cairo Museum. Image in the public domain.

The Lincoln Cathedral with its west tower spires became the tallest structure in the world in 1300 A.D. Print from the 17th century.

Background image –The Great Pyramid photographed by Jorge Láscar; top frame the stones of the Great Pyramid photographed by Jon Bodsworth.

Numbers of Materials & Stone Cutting

Estimated Number of Materials Used:

- 5.5 million tons of limestone for outer pyramid casing
- 8,000 tons of granite (imported from Aswan) for core
- 500,000 tons of mortar

Traditional stone cutting belief:

- Workers hammered into stone with wooden wedges
- Wedges soaked with water
- As water absorbed, wedges expanded, causing rock to crack

Casing stone in the British Museum photographed by CaptMondo.

The Great Pyramid photographed by Jerome Bon.

The Great Pyramid's Building Materials & Transportation

- Estimated 2.3 million blocks used
- Blocks probably transported from nearby quarries
- Limestone casing quarried across river from Tura, Egypt
- Largest granite stones - found in "King's" chamber, weigh 25 to 80 tons; transported from Aswan, more than 800 km (500 mi) away
- Cut blocks carried by boat either up or down Nile River to pyramid

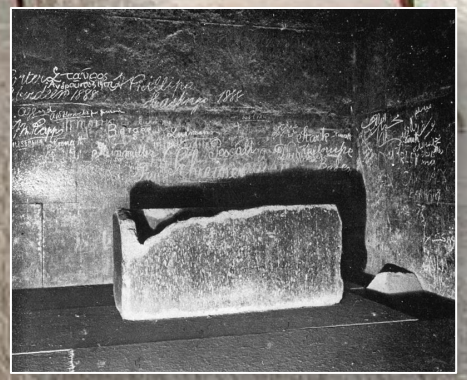
The Royal Cubit (side bar left) was used in the Great Pyramid's building measurements. Sidebar cubit rod photographed by Bakha.

Background image – Stone blocks from the Great Pyramid photographed by Bs0u10e0.

Great Pyramid's Chamber Use

- First chamber - underground, carved into bedrock
- Second chamber – above ground called in error "queen's chamber" by early explorers (she has her own burial pyramid); used, perhaps for sacred statue of king himself
- Third chamber – assumed to be for king's burial, with one red granite sarcophagus placed almost exactly at pyramid center
- King's chamber access – via 26-foot-high (8-meter-high) Grand Gallery sealed off from thieves by sliding granite blocking systems

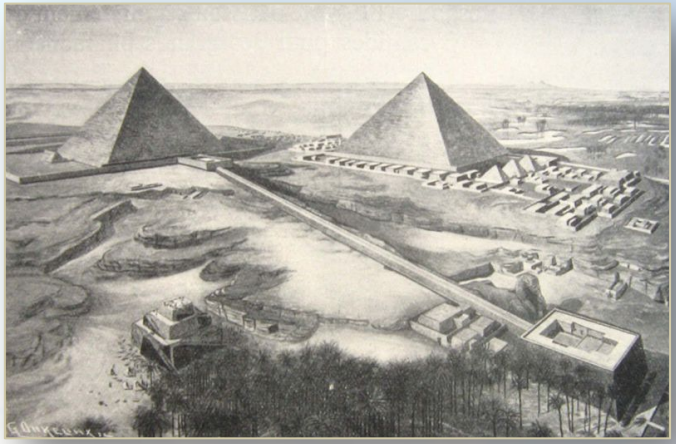
Background image -The Al-mamoun-tunnel in the Great Pyramid photographed by Jon Bodsworth.



Sarcophagus in the King's chamber photograph from the Brooklyn Museum.

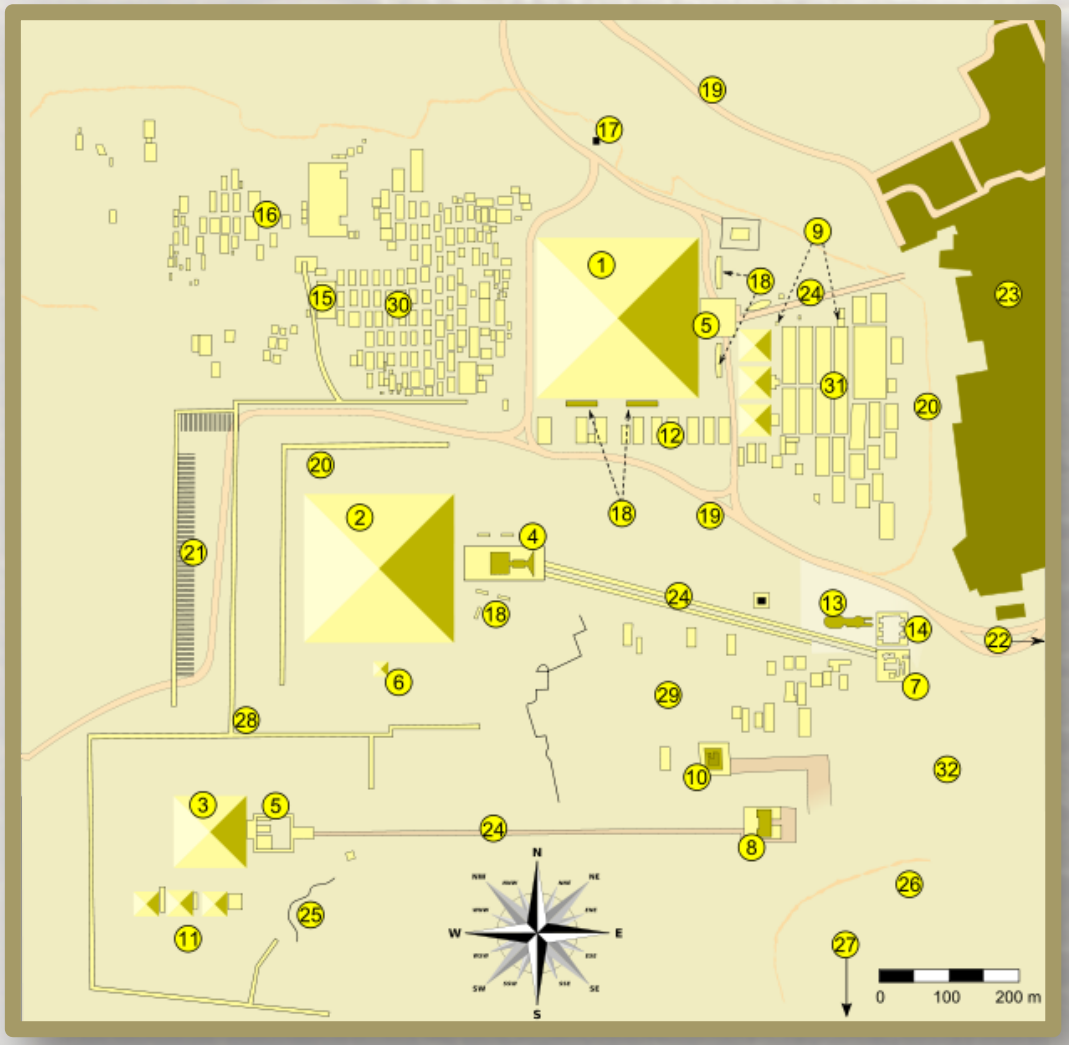
The Great Pyramid Complex

- Centerpiece - Great Pyramid
- Several smaller pyramids
- 5 boat pits
- A mortuary temple
- A causeway
- A valley temple
- Many flat-roofed tombs for officials & royal family members



Images – All of the Giza Pyramid complex; background photographed by Wknight94; upper border by Kallerna and smaller image photographed in 1910 by Uvo Holscher.

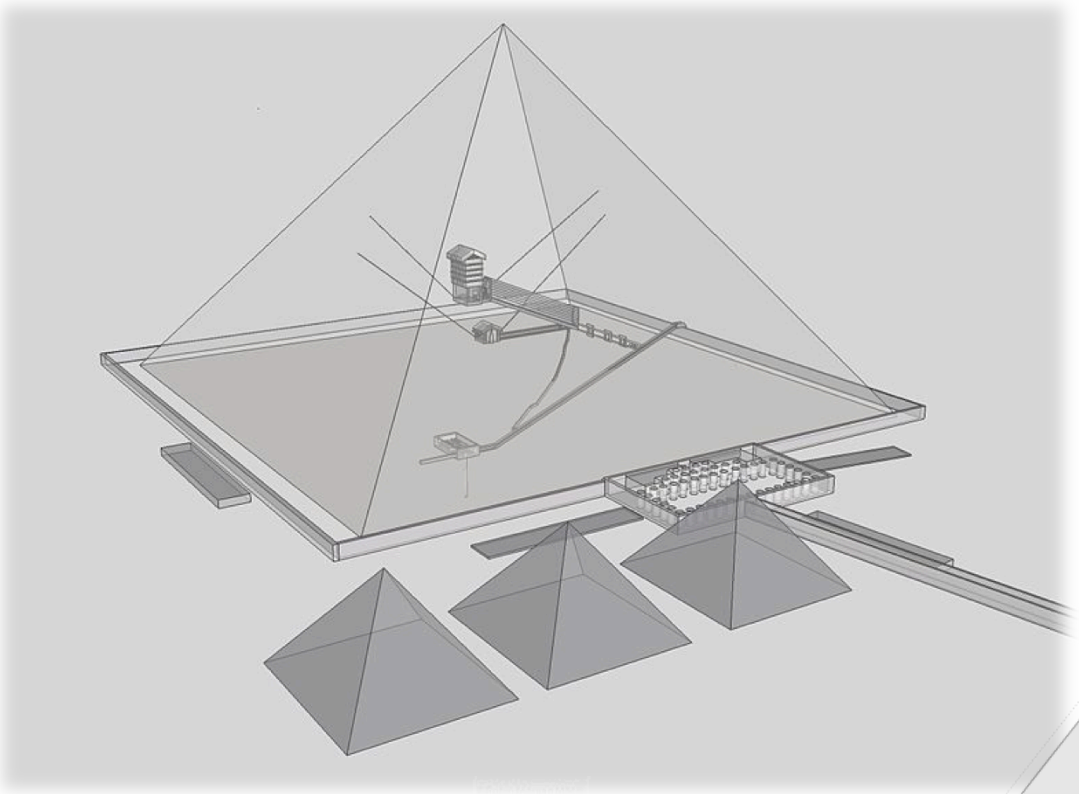
Map of the Pyramids of Giza Complex



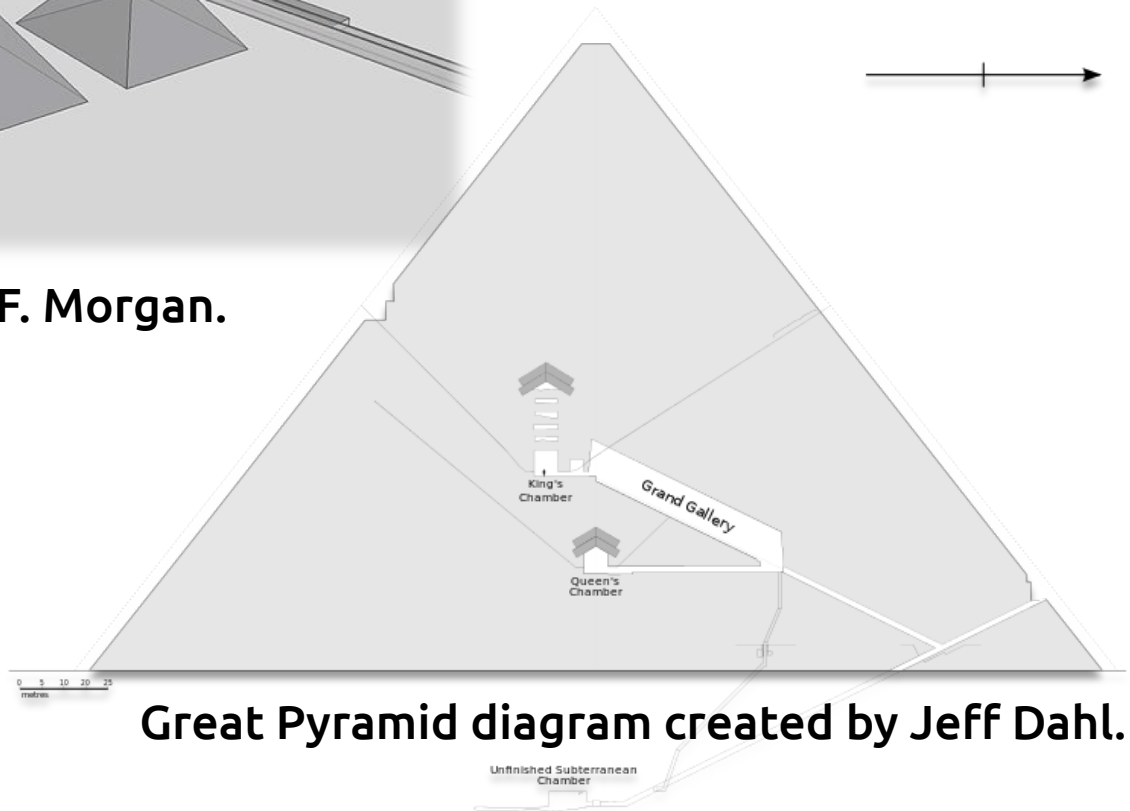
1. Pyramid of Khufu
2. Pyramid of Khafre
3. Pyramid of Menkaure
4. Funerary Temple of Khafre
5. Funerary Temple
6. Subsidiary pyramid
7. Valley Temple of Khafre
8. Valley Temple of Menkaure
9. Queen Hetepheres's tombs
10. Tomb of Queen Khentkawes
11. Pyramids of Queens
12. Mastabas
13. Great Sphinx
14. Temple of the Sphinx
15. Tomb of Hemon
16. Office of pyramids studies
17. Ticket office
18. Boat pits
19. Modern road
20. Rock cut tombs
21. Builders' quarters
22. Cairo
23. Village of Nazlet el-Samman
24. Causeway
25. Menkaure quarry
26. Modern cemetery
27. Southern field of mainly rock-cut tombs
28. Enclosure wall
29. Mastabas and rock-cut tombs
30. Western cemetery
31. Eastern cemetery
32. Central field of Mastabas and rock-cut tombs

Images: map by Messer Woland; top frame and background transparency photographed by Captmondo.

Inside the Great Pyramid



3-D view created by R.F. Morgan.



Great Pyramid diagram created by Jeff Dahl.

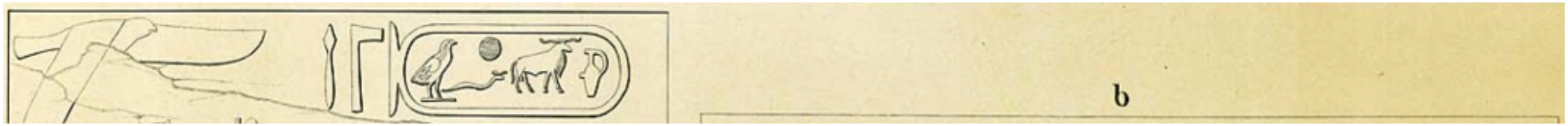
Please note: The following mathematics are debated and are yet considered theories in many archaeological, scientific and mathematics communities. This is mainly because the Great Pyramid's outer stone casing has been removed making the original dimensions more challenging to measure today.

The Great Pyramid & the Golden Ratio

Evidence reveals the following mathematics are found in the Great Pyramid. Each will be addressed on the following slides:

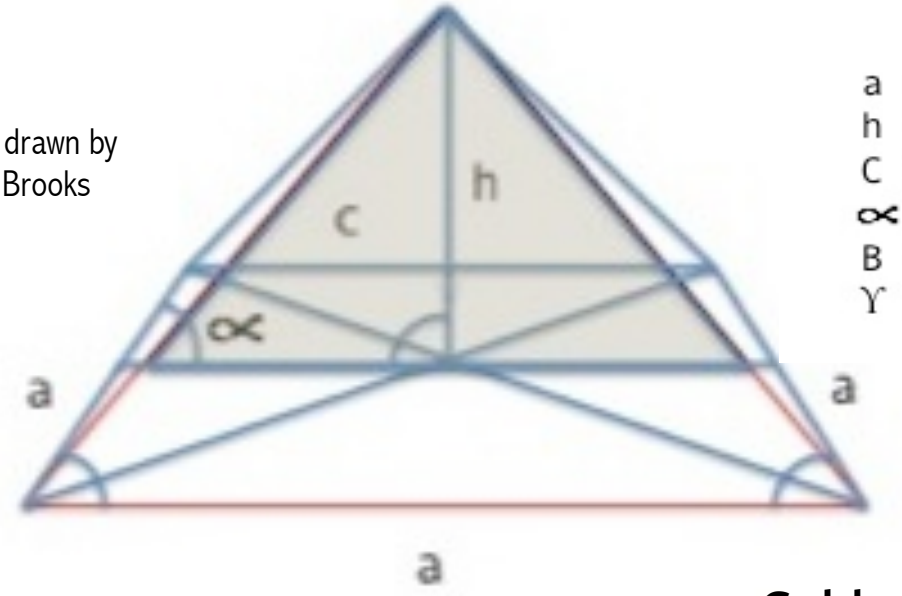
- Phi, or Golden Ratio; appears throughout nature
- Pi - circumference of a circle in relation to its diameter
- Pythagorean Theorem – Founded by Pythagoras (about 570 – 495 BC) with formula as $a^2 + b^2 = c^2$

Background image - *Pyramids of Gizah* painted by David Roberts (1796 – 1864)



Dimensions of the Great Pyramid in Royal Cubits

Diagram drawn by
Gloria A. Brooks



a = 440 cubits
h = 280 cubits
C = 356 cubits
∝ = 51.8 degrees
B = 58.3 degrees
ϒ = 41.9 degrees

- Golden ratio relationship to Great pyramid revealed in Egyptian royal cubit dimensions
- Egyptians may have used right angled triangle to determinate pyramid's angle of inclination

The Rhind Mathematical Papyrus is the best record of Egyptian mathematics. It is in several parts equaling 16 feet wide and 13 inches long.

Top frame image – Relief of Khufu from Valley of the Caves in the public domain.

Review of the Golden Ratio or Phi

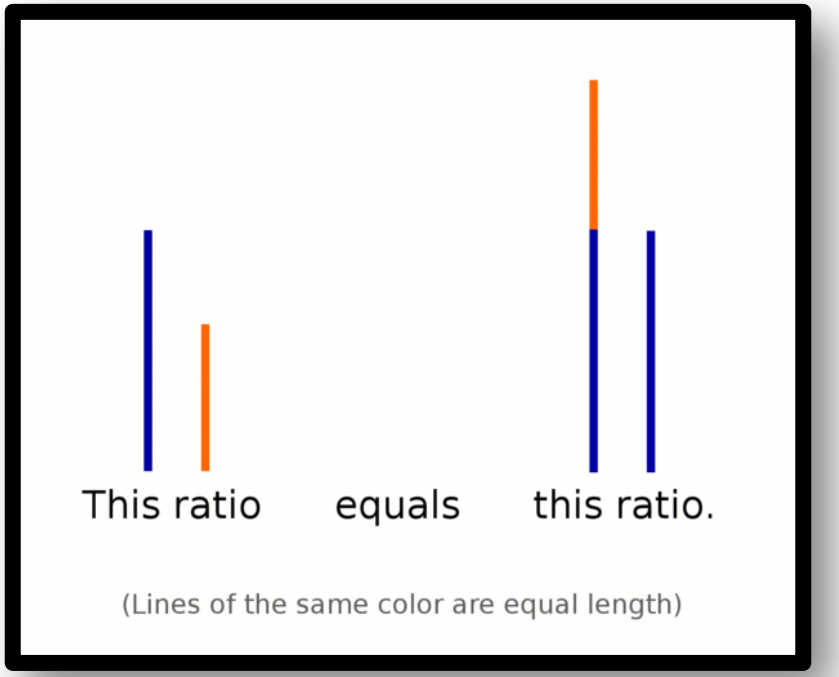
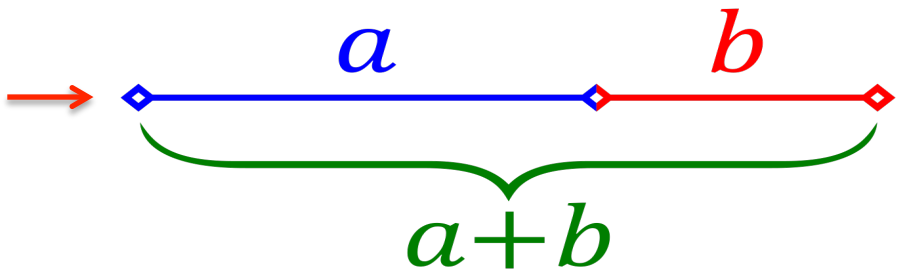


Diagram by Sparkie82

- Other names – several including Phi, golden ratio, golden section, golden mean, divine proportion
- Ratio - an irrational, infinite number: 1:1.61803398874989485…
- Found in - art, architecture, design, nature
- Ratio used throughout history and today for aesthetic beauty

- Line divided into 2 segments
- 2 segments - *a* and *b*: entire line is to longer *a* segment as *a* segment is to shorter *b* segment



$a+b$ is to a as a is to b

What is Pi or π?

What is Pi?

- **Ratio of a circle's dimensions**
- **Circumference ÷ diameter = 3.1415926535897932…etc.**
- **Pi and Phi - irrational numbers**

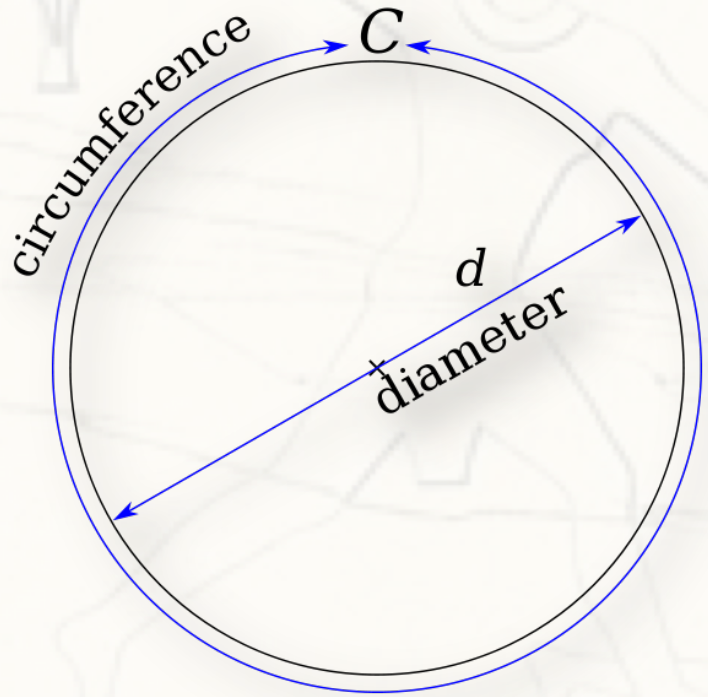


Image in the public domain.

What are irrational numbers?

- **Cannot be fractions**
- **Infinite number of digits to decimal**
- **Don't end with infinitely repeating digit patterns**

Images - Background: Carvings of Khufu from Valley of Caves in the public domain. Side bar frame of the Great Pyramid's blocks by Bs0u10e0.

The Great Pyramid, a Golden Pyramid?

- Phi ratio also found in triangle formed by height, half-base, and apothem, or diagonal
- Basic cross-section demonstrates golden ratio

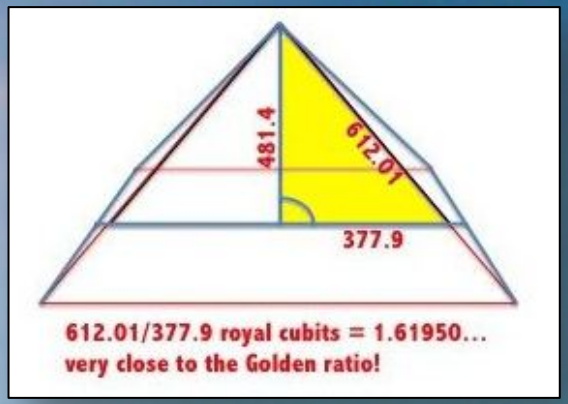


Diagram created by Gloria Brooks.

Images – Background: Sun over the Great Pyramid photographed by Kheops. Upper border: Relief from Ramses II Temple photographed by JMCC1.

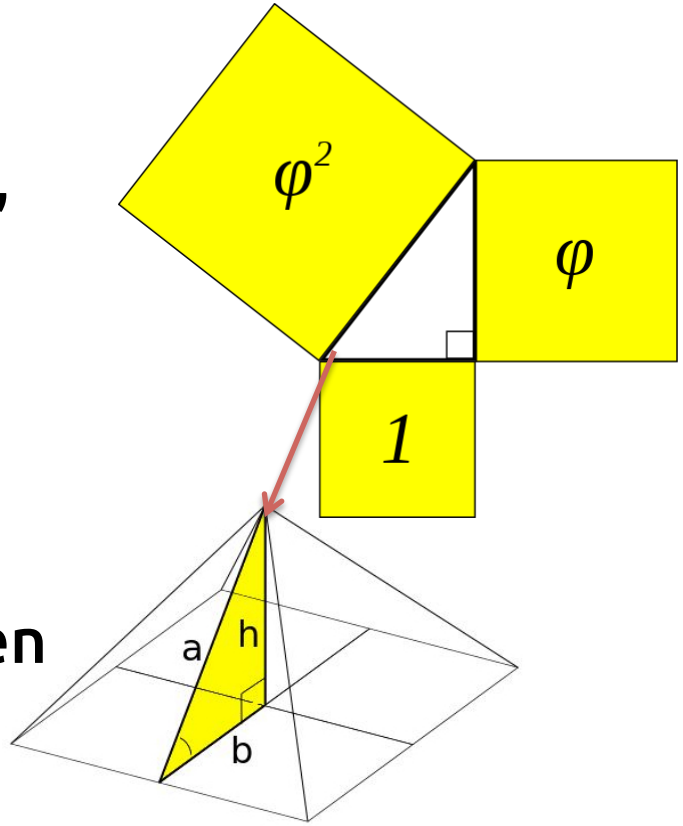
- Pyramid base - measures 755.8 cubits; half being 377.9 cubits
- When divided by length of one of its inclined sides (612 cubits), result is 1.619
- 1.619 very close to divine ratio

Great Pyramid image photographed by Kheops.

The Great Pyramid & the Golden Triangle

The Kepler or Golden triangle:

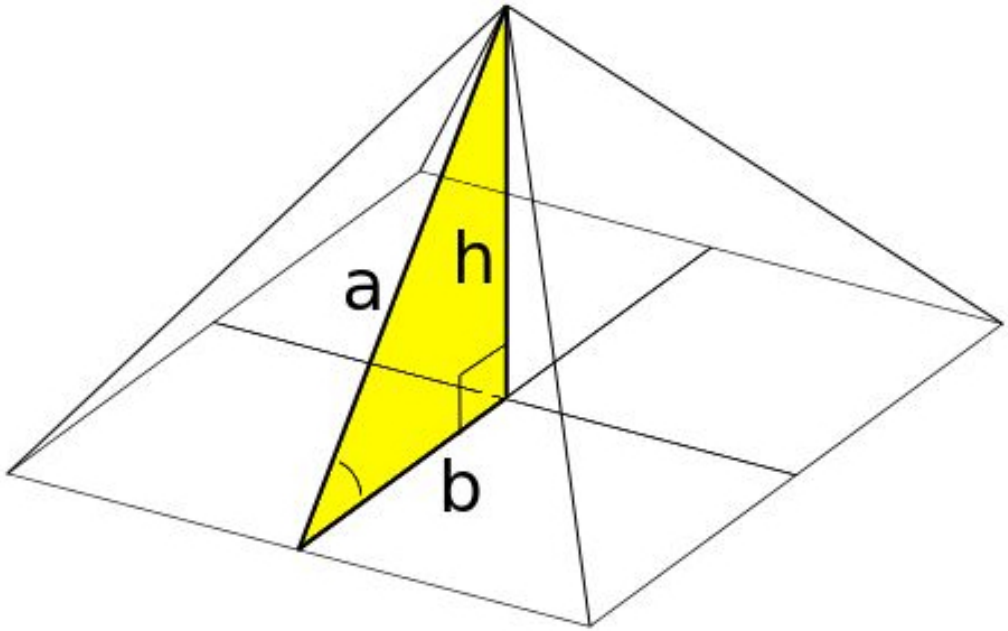
- Johannes Kepler (1571–1630), astronomer, wrote about it
- A right or golden triangle found in Great pyramid
- Formed by three golden ratio squares or triangle's square edges (see figure) make golden ratio



Images: Pyramid image by by Dicklyon Creative Commons license & derivative shading work by Gloria Brooks. Top frame of Sphinx's head by Barcex.

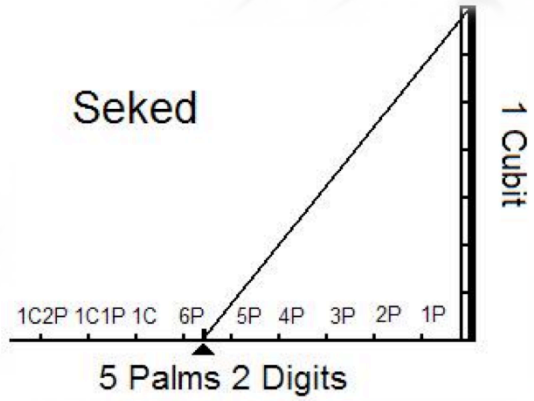
The Great Pyramid & the Golden Ratio

- Has *b* (base) of length 1 + Phi (a) = Phi ², or height (*h*) of right triangle is square root of phi (*a*)
- Less than 0.025% from perfect golden triangle pyramid
- Using Pythagorean theorem ($a^2+b^2=c^2$), this triangle represents one of golden ratio's unique properties or 1 + Phi = Phi ²

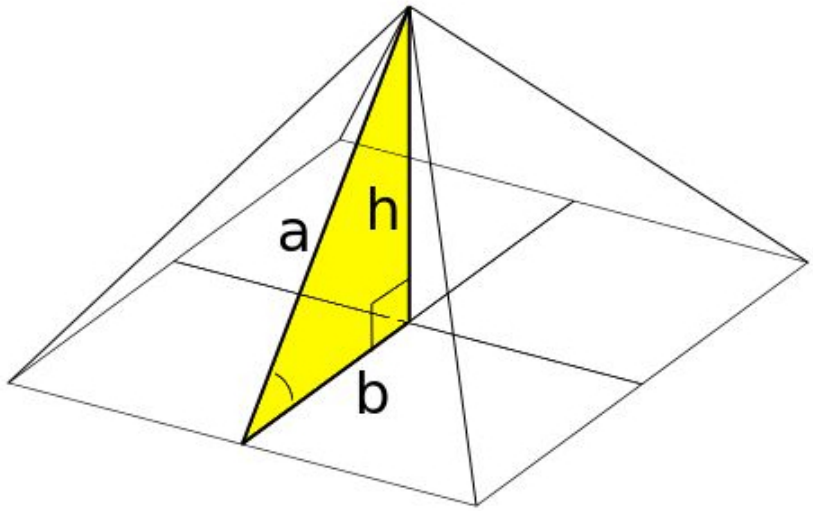


The Kepler Triangle & the Pi Based Great Pyramid

- Regular square pyramid determined by its medial right triangle (Kepler triangle)
- Edges (diagram lower right) - pyramid's apothem (a), semi-base (b), and height (h)
- Mathematical proportions equal the golden number 1.61899…
- Slope of 51° 52’; extremely close to "golden" pyramid inclination of 51° 50’



The Egyptian Seked was a measurement unit likely used for finding the Great Pyramid’s slope. On the Seked, it is based on 5 palms and 2 digits.



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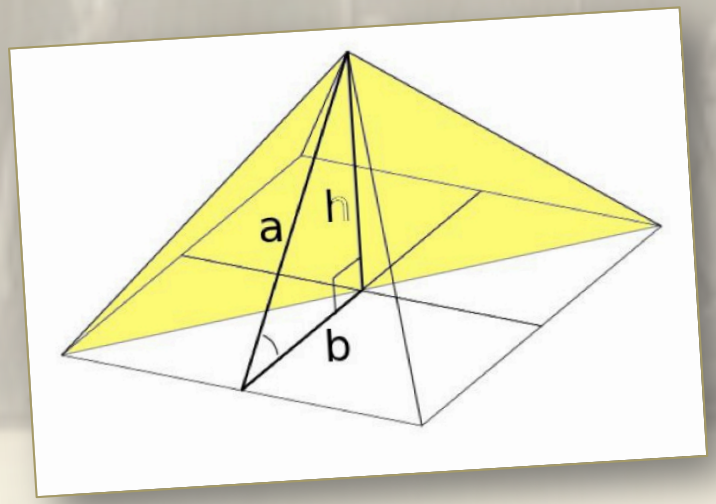
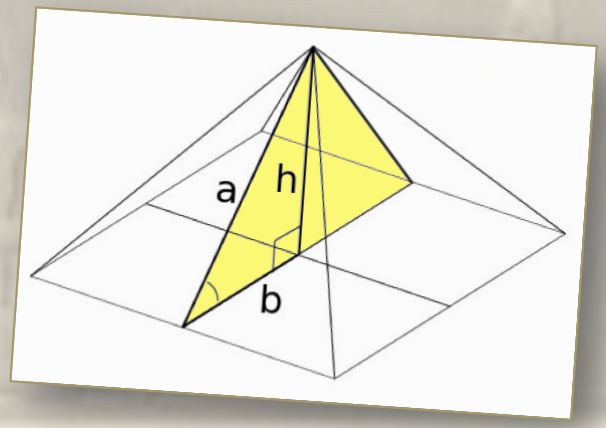
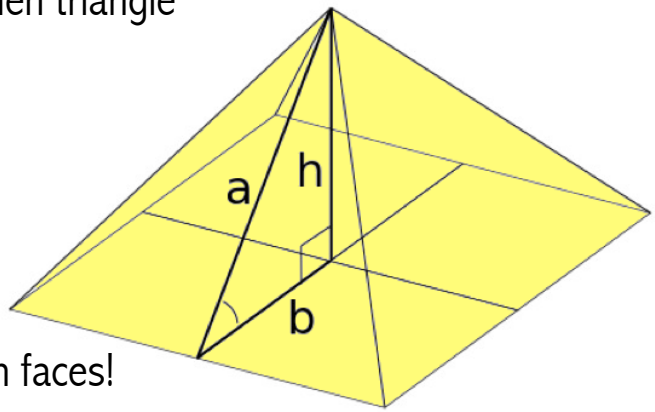
The Great Pyramid, A Golden Pyramid?



The Great pyramid's diagonal golden triangle



A golden perpendicular triangle



All golden faces!

Pyramid images by Dicklyon, Creative Commons license with derivative shading work by Gloria Brooks.

Top frame image of Egyptian hieroglyphics carved into stone and background transparency of relief stone carvings from the book, *Description of Egypt* from the late 17th century.. Images in the public domain.

The Rope Stretcher's Triangle, or the 3-4-5 Triangle with use of the Pythagorean Theorem

- Other names - Rope-Knotter's triangle, Pythagorean triangle
- Evidence shows Egyptians used rope knotted into 12 sections stretched to form 3-4-5 triangle
- Can it make a right angle?
- Was it used in building the Great pyramid?

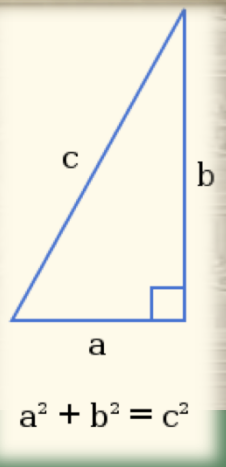


Image by Pythagoras abc.png: nl:Gebruiker: Andre_Engels

The Pythagorean theorem states:

- "In a right triangle, the square of the hypotenuse equals the sum of the squares of the legs."

The converse of is also true:

- "If the square of one side of a triangle equals the sum of the squares of the other two sides, then we have a right triangle."

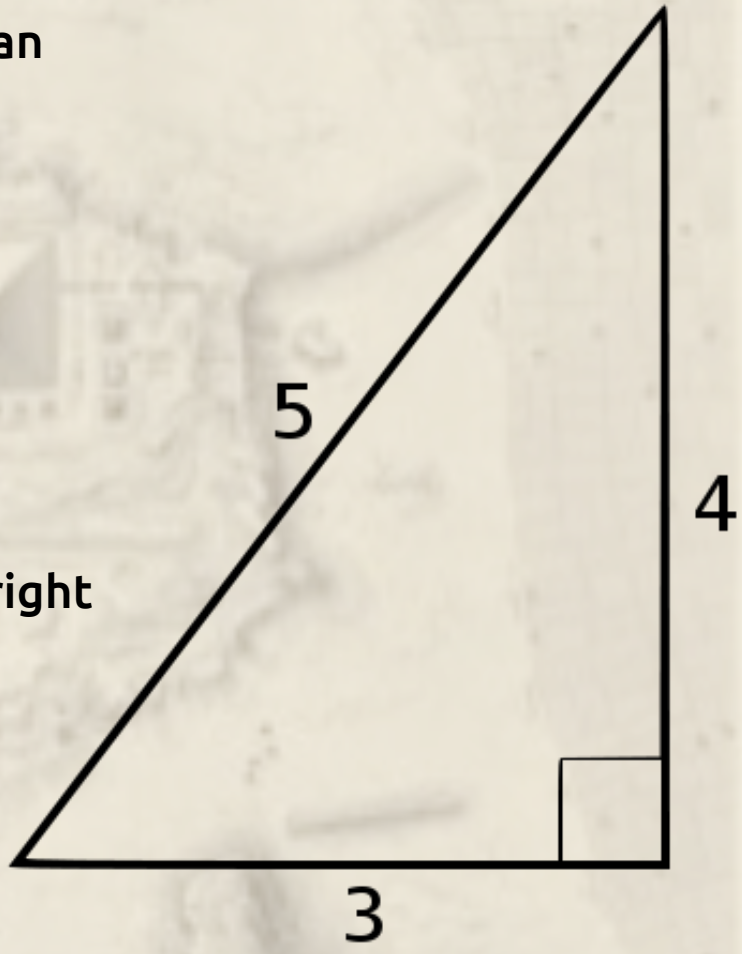
The 3-4-5 Triangle Continued

3-4-5 Triangle useful for determining if an angle is a right angle:

$$5^2 = 3^2 + 4^2$$

$$25 = 9 + 16$$

- **It checks!**
- **Reveals rope knotted this way gives right angle**



Public domain image.

Background transparency & thumbnail of the Pyramid complex from the book, Description of Egypt and in the public domain.

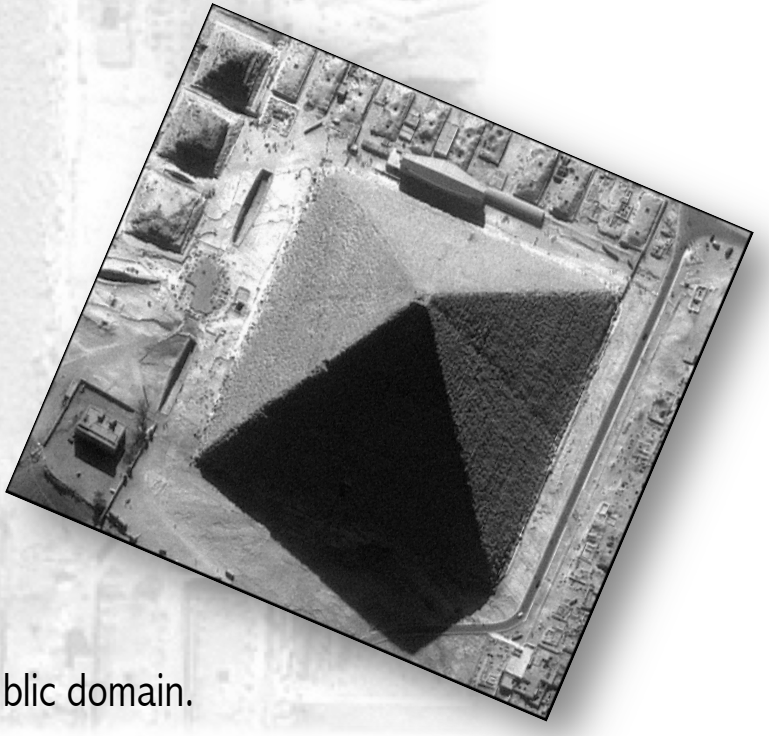
The Great Pyramid's Concave Faces

- 4 faces precisely concave & hollowed to inside
- An 8-sided pyramid, rather than 4-sided like most pyramids
- Invisible on ground; only noticed from air

- March 21, 1940 - British air force pilot flying over Giza Plateau discovered & photographed

Possible concavity uses include:

- Prevent casing stones from sliding; better bond nucleus to casing
- Aesthetic one: concave faces pleasing to view



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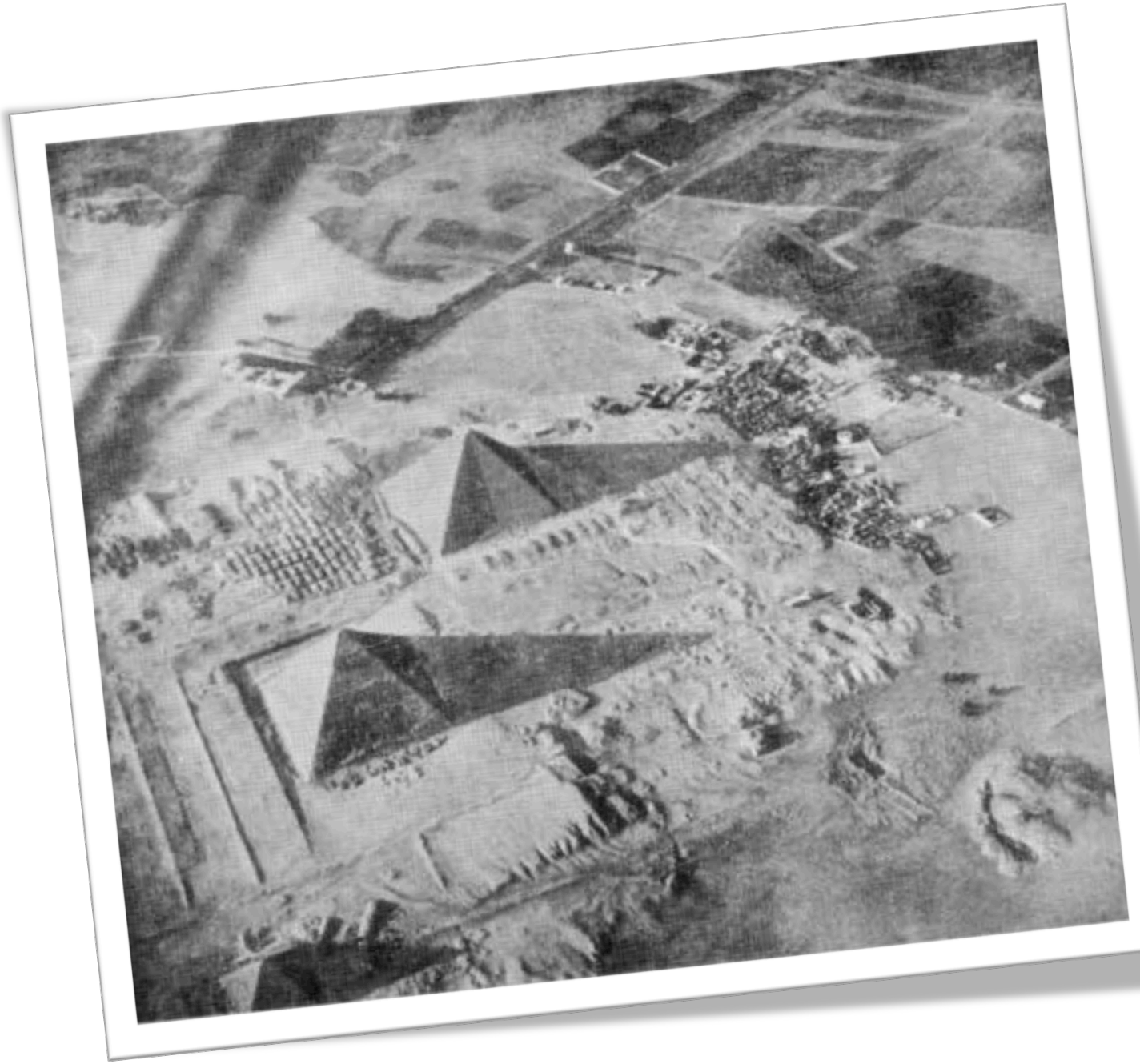
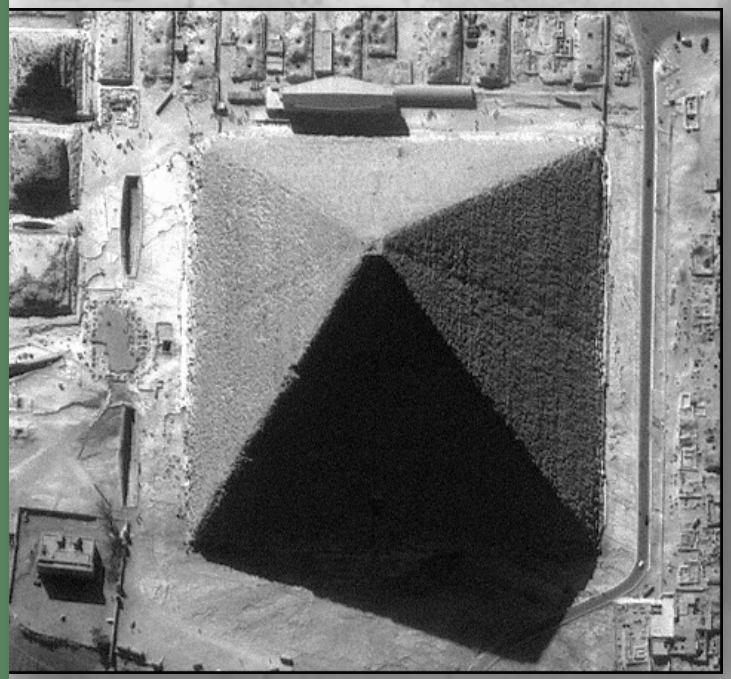


Enlarged aerial view revealing the concavity of the Great pyramid. Look for and point at the Great pyramid's revealed concave side (all four sides are concave). Image in the public domain.

Great Pyramid's Concave Faces & Phi

- Value of Phi, equal to secant of face angle of inner triangle, same as ratio between slant height of pyramid and half its base-length



- Phi applies to all 8 right-angle triangular faces
- Each 8-sided face confirms with golden ratio proportion: it is inscribed into golden rectangle where height/ base = Phi


Learn more about the Golden ratio and the Great pyramid by visiting NatureGlo's eScience MathArt Virtual Library at the link below.
http://hascmathart.weebly.com/the-great-pyramid.html
The Great Pyramid's northeast view photographed by Jon Bodsworth Copyright Free Use.

References

1. National Geographic - http://www.nationalgeographic.com/pyramids/khufu.html
2. World Mysteries: The Golden Ratio & Squaring of the Circle of the Great Pyramid: http://www.world-mysteries.com/mpl_2.htm#Socrates
3. Wikipedia – Golden Ratio article: https://en.wikipedia.org/wiki/Golden_ratio
4. Pyramid Geometry - https://pyramidgeometry.wordpress.com/
5. Dartmouth College - https://www.dartmouth.edu/~matc/math5.geometry/unit2/unit2.html

Egyptian hieroglyphics photographed by by Michael Holford.

Thank you for watching!

Image of the Sphinx photographed by Barcex.