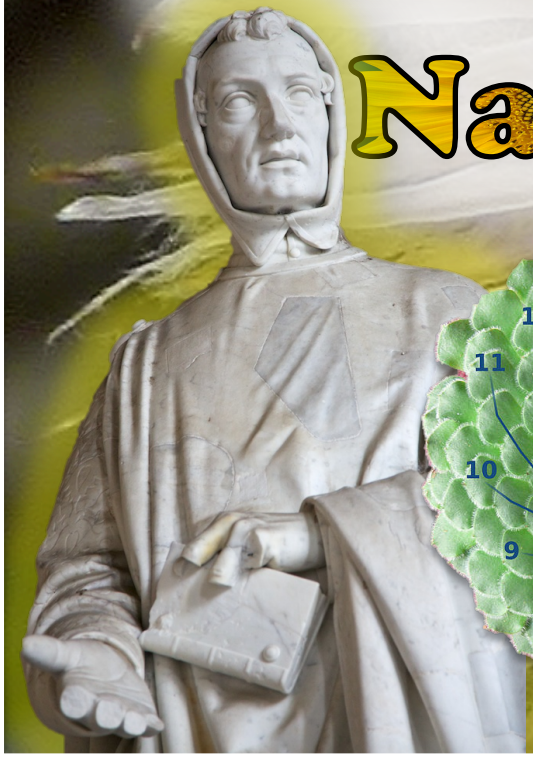
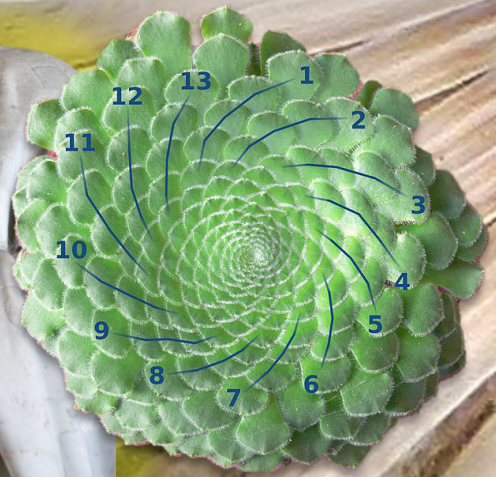
NatureGlo's eScience

The Fibonacci Numbers in Nature

1, 1, 2, 3, 5, 8, 13, 21, 34, 55





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Cover image – Background image Marlborough Rock Daisy photographed By Sid Mosdell. The statue of Fibonacci photographed by George Grinsted. The green spiraling plant (including this slide's background), *Aeonium tabuliforium* photographed by Peter. The apple cross-section revealing a 5-pointed star, photographed by Rasback.

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What are the Fibonacci Numbers?

Statue of Leonardo of Pisa, or "Fibonacci" photographed by George

- Sequence of infinite numbers
- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55….etc. into infinity
- Numbers frequently found in nature
- Named after Leonardo of Pisa (1170-75 – 1240-50 AD)
- Leonardo of Pisa is usually called "Fibonacci"

Who was Leonardo of Pisa?

Portrait of Fibonacci by unknown medieval artist.

- 1170 – 1250 A.D. - mathematician born in Pisa, Italy
- Other names of Leonardo - Leonardo Bonacci, (known as Fibonacci), Leonardo of Pisa, Leonardo Pisano Bigollo, Leonardo Fibonacci
- Considered "most talented Western mathematician of the Middle Ages"

The Leaning Tower of Pisa photographed by

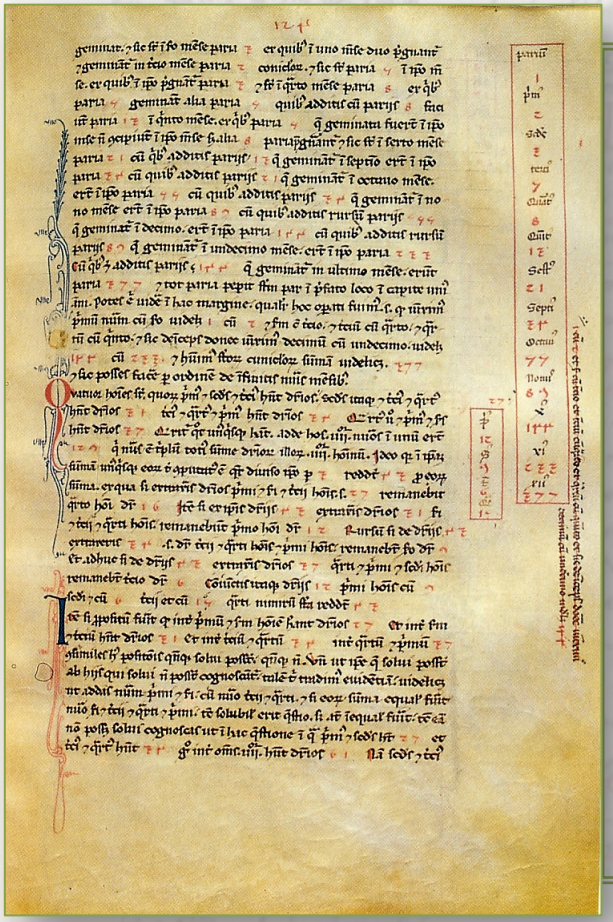
A Brief History of Fibonacci Numbers

- 1202 - Popularized Hindu–Arabic numeral system (numbers 0 and 1 – 9) to Western World mainly through his book, *Liber Abaci* (Book of Calculation)
- *Liber Abaci* introduced Fibonacci numbers sequence to Europe
- 19th century - statue of Fibonacci constructed & erected in Pisa, Italy

Full statue view of Fibonacci photographed by Hans-Peter Postel.

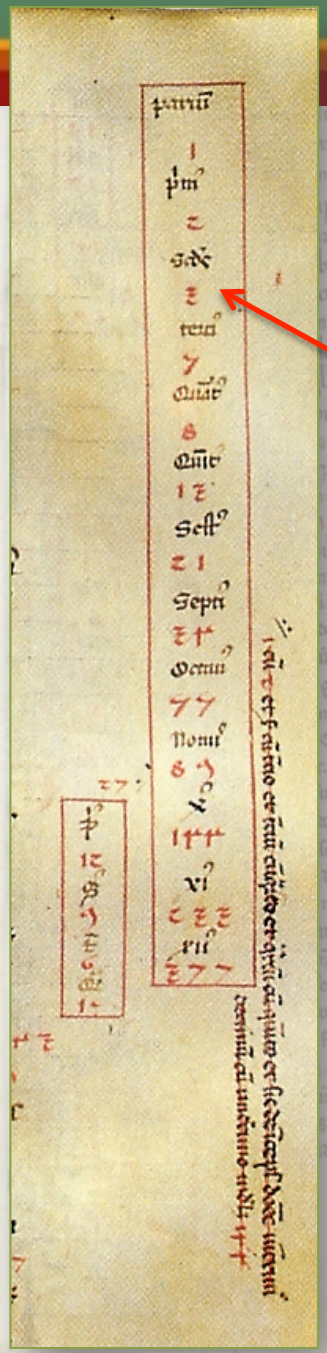
Background image – Sunflower showing two spiral turns. The spiral turns in each direction, most likely yield Fibonacci numbers. Image by Esdras Calderan.

What's in Fibonacci's book, *Liber Abaci*?



- Introduced Hindu-Arabic numerals (0, 1 – 9) & place value to Europe
- 0, 1 - 9 applied to commercial bookkeeping, weights conversion, measures, interest calculation, money-changing & more
- Reveals Fibonacci sequence

Background transparency and Image from pages of the *Liber Abaci*. Images in the public domain.



Historical Significance of *Liber Abaci*

- Fibonacci number sequence labeled in Roman numerals
- Value shown in Hindu-Arabic numerals
- Book showed practical importance of new numeral system
- Book well received throughout Europe which impacted European mathematical thought & practice

Background image — A 2-page spread of the Liber Abaci. Image in the public domain.

Fibonacci Numbers & its Relationship with Phi

- Starting with 1 + 1 (or 0 + 1) each new number in sequence is sum of two previous numbers
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610
- Ratios of successive Fibonacci number sequence gets closer to Phi

Sidebar image of an apple's 5-pointed star by Rasback; background transparency showing sunflower spirals created by David Convent.

Fibonacci Numbers Approach & Become Phi!

- By dividing a Fibonacci number by number before it, by 12th digit (89) in sequence, approaches golden ratio 1.618034…etc. & gets closer & closer to exact phi sequence
- See examples below in green box
- After 40th number, ratio is accurate to 15 decimal places to phi

A vintage painting of a wild artichoke by Ferdinand Bernhard Vietz. 1817.

1/1 = 1, 2/1 = 2, 3/2 = 1·5, 5/3 = 1·666…, 8/5 = 1·6, 13/8 = 1·625, 21/13 = 1·61538… …

The Use of Fibonacci Numbers, Past & Present

- 6th century AD – Fibonacci sequence used in ancient India for metrical sciences, or as prosody (poetic meter study)
- Used by Fibonacci to illustrate idealization rabbit population growth
- Used also in idealized cow (Dudney cows) and honey bee populations (honey bees & family trees)

An artichoke from Pixabay and in the public domain.

Background image – Pineapples photographed by Ramon F. Velasquez.

More Fibonacci Numbers Found in Nature

- Two consecutive Fibonacci numbers found in tree branches
- Numbers of leaves on stem can be Fibonacci numbers
- Pinecone, pineapples & artichokes scale numbers are usually Fibonacci numbers
- By constructing rectangle set in spiral formation using Fibonacci numbers as unit lengths, resulting spiral very similar to snail, nautilus, and other shells spirals

Background image – An artichoke photographed by Clairefitton.

Fibonacci Phyllotaxis

Leaf pattern arrangements or lattice types:

- Phyllotaxis definition – plant leaf arrangements on a stem
- Spiral - most frequent; classified by number of spirals (parastichies) exhibited
- Distichous - (of parts) arranged alternately in two opposite vertical rows

Distichous leaf (background transparency) arrangement in *Clivia* or Bush lily. Photographed by Narutodude000

Alternate spiral leaf pattern CC BY-SA 3.0.

Alternate spiral leaf pattern by Cayte.

Whorled leaf pattern CC BY-SA 3.0.

Sidebar frame, Vincent Van Gogh painting, Three Sunflowers 1888. Image in the public domain.

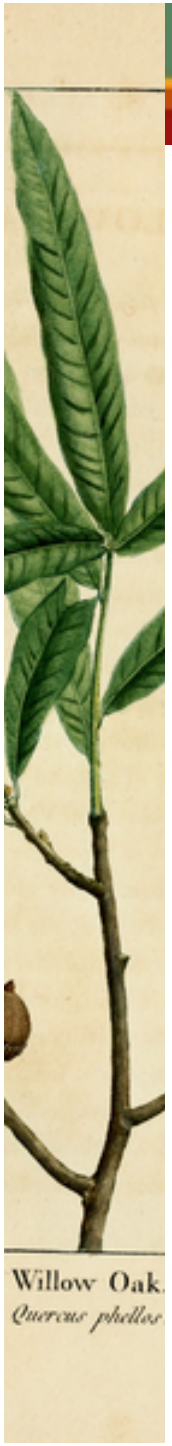
Fibonacci Phyllotaxis – Whorled, Multijugate, Fibonacci Numbers & the Golden Angle

Whorled leaves of Wild or Bitter Almond tree by Abu Shawka.

- Whorled – radiate from single point; surround or wrap around stem
- Multijugate - two or more leaf primordia grow at same node and spread evenly around stem
- Number of visible spirals (parastichies) in spiral arrangements most often Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, 21 ...)
- Angle between successive leaves close to Golden Angle - about 137.5 degrees

Whorling leaves of the Michigan Lily photographed by Douglas W. Jones.

Bottom frame of an aloe plant photographed by Stan Shebs.



Fibonacci Phyllotaxis: Repeating Spirals & their Leaf Angles

Represented by a fraction describing leaf windings angle per leaf:

- Alternate distichous - 1/2 of a full rotation
- Beech and hazel - 1/3
- Oak and apricot - 2/5
- Sunflowers, poplar, & pear - 3/8
- Willow and almond - 5/13
- Numerator and denominator usually consist of Fibonacci number and its second successor

Willow oak leaves (an oak). Public domain.

Copper beech tree leaves photographed by David Hawgood.

Background image – Silhouette of an oak tree leaf. Image by Spedona.

Phyllotaxis in Art and Architecture

- Phyllotaxis - used as inspiration for many sculptures and architectural designs
- Akio Hizume - has built and exhibited several bamboo towers based on Fibonacci numbers which exhibit phyllotaxis
- Spiral apartment building – proposed by Saleh Masoumi ; design where spirally designed apartment balconies project around central axis; each balcony wouldn't shade balcony of apartment directly beneath

Background & thumbnail
The Weizmann House
spiral staircase
photographed by Ovedc.

Fibonacci Numbers in Plants

A fuchsia daisy from Pixabay and in the public domain.

Numbers of flower parts (mostly petals):

- 3 - iris, lily, trillium
- 5 - buttercup, columbine (aquilegia), larkspur, pinks, wild rose
- Cultivated buttercups bred into multi-petal forms
- 8 – delphiniums, some daisy cultivars
- 13 - corn marigold, cineraria, daisies (some) ragwort

Buttercup by Laura Brolis.

Painted trillium photographed by T.G. Barnes and in the public domain.

A 13-petaled ragwort by Jeantosti.

Background image – close-up of a red rose from Pixabay and in the public domain.

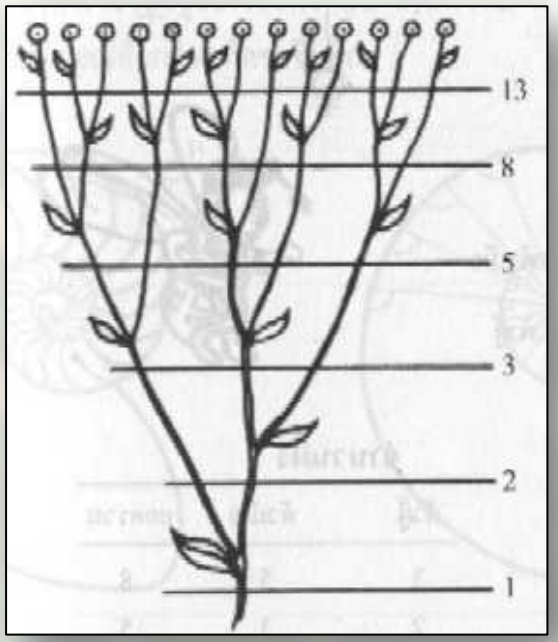
Fibonacci Flowers 21 - 89

- 21 - aster, Black-eyed Susan, chicory
- 34 - plantain, pyrethrum
- 55, 89 - Asteraceae family, michaelmas daisies, roses
- Some species precise about number of petals such as wild buttercups; others with varying petal numbers; average being Fibonacci numbers

Background image – Count the petals on the Black-eyed Susan in the background. How many petals does it have? Is it a Fibonacci number? Answer on the next slides

Fibonacci Numbers in the Sneezewort Plant

- New branches usually grow out at axil, space between a leaf or branch and stem which it's attached
- A horizontal line through each axil shows growth stages (image right)
- Main stem produces new branches at beginning of each growth stage



Sneezewort showing a Fibonacci growth sequence.

Background transparency – Sneezewort photographed by Alan Fryer.
Answer from previous slide – the Black-eyed Susan has 13 petals. 13 is a Fibonacci number.

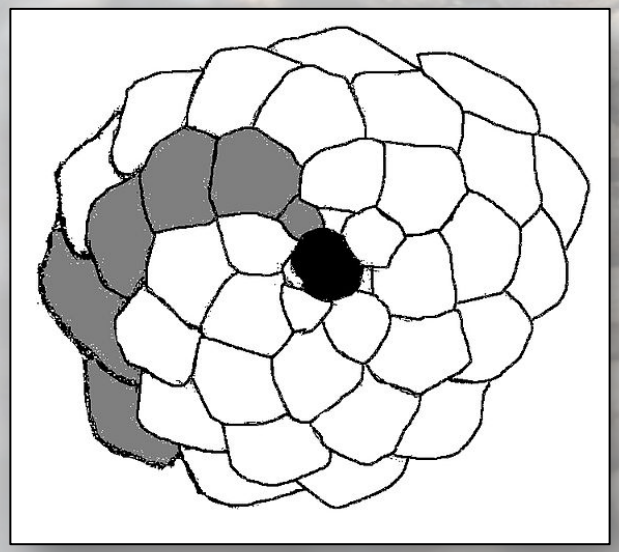
Fibonacci & the Sneezewort Plant

- New branches rest for two stages of growth, then begin producing new shoots at beginning of each growth stage
- New growth shoot numbers at any given stage will be a Fibonacci number
- Leaves and branches generally grow in spiral pattern around main stem

Wild Sneezewort photographed by Lairich Rig.

Fibonacci Numbers in Pinecones

The number of spiral turns about the pinecone, both clockwise and counterclockwise, are most often Fibonacci numbers. Count the turns using the starting points of the black and white example. Is it a Fibonacci number?



Created by MathPics and in the Universal Public Domain Dedication.

Image by Jean-Luc W

A female Pinus coulteri cone photographed by Didier Descouens.

Background image of a pinecone photographed by Ben Klocek.

Pineapple Fibonacci Exhibit

Pineapples show left- and right handed spirals, usually revealing Fibonacci numbers. The ratio of the high-slope versus the low-slope helix approximates the golden ratio: (3/2, 5/3, 8/5, 13/8).

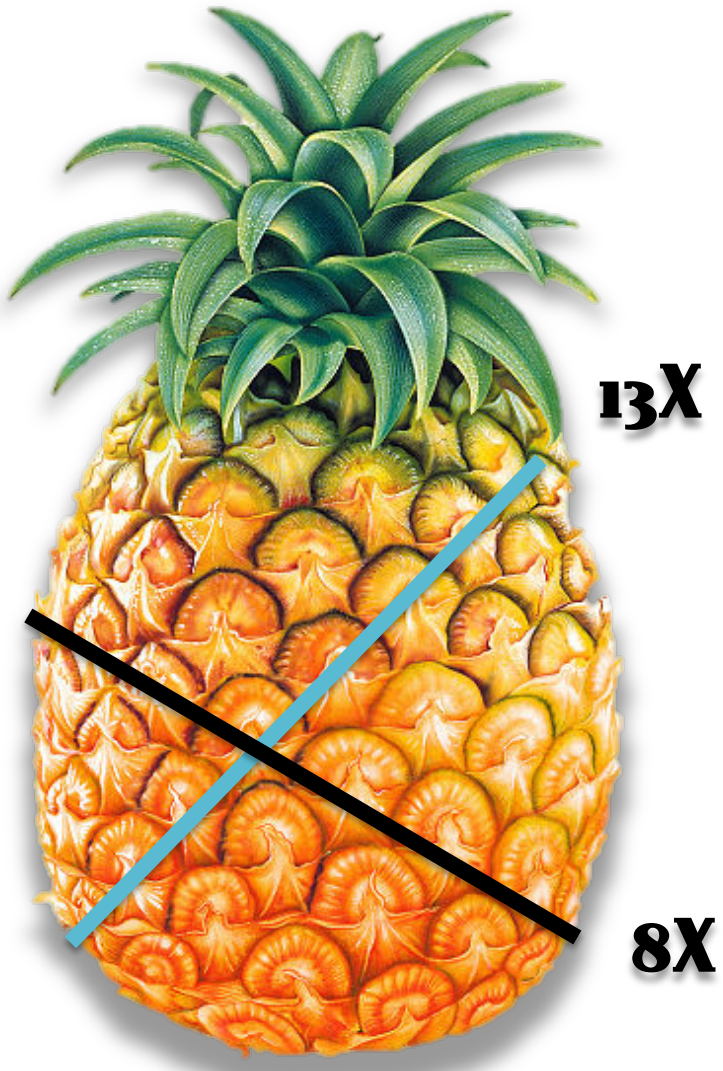


Image by NTGala4

What Fibonacci number do you notice with these apple horizontal cross-sections?

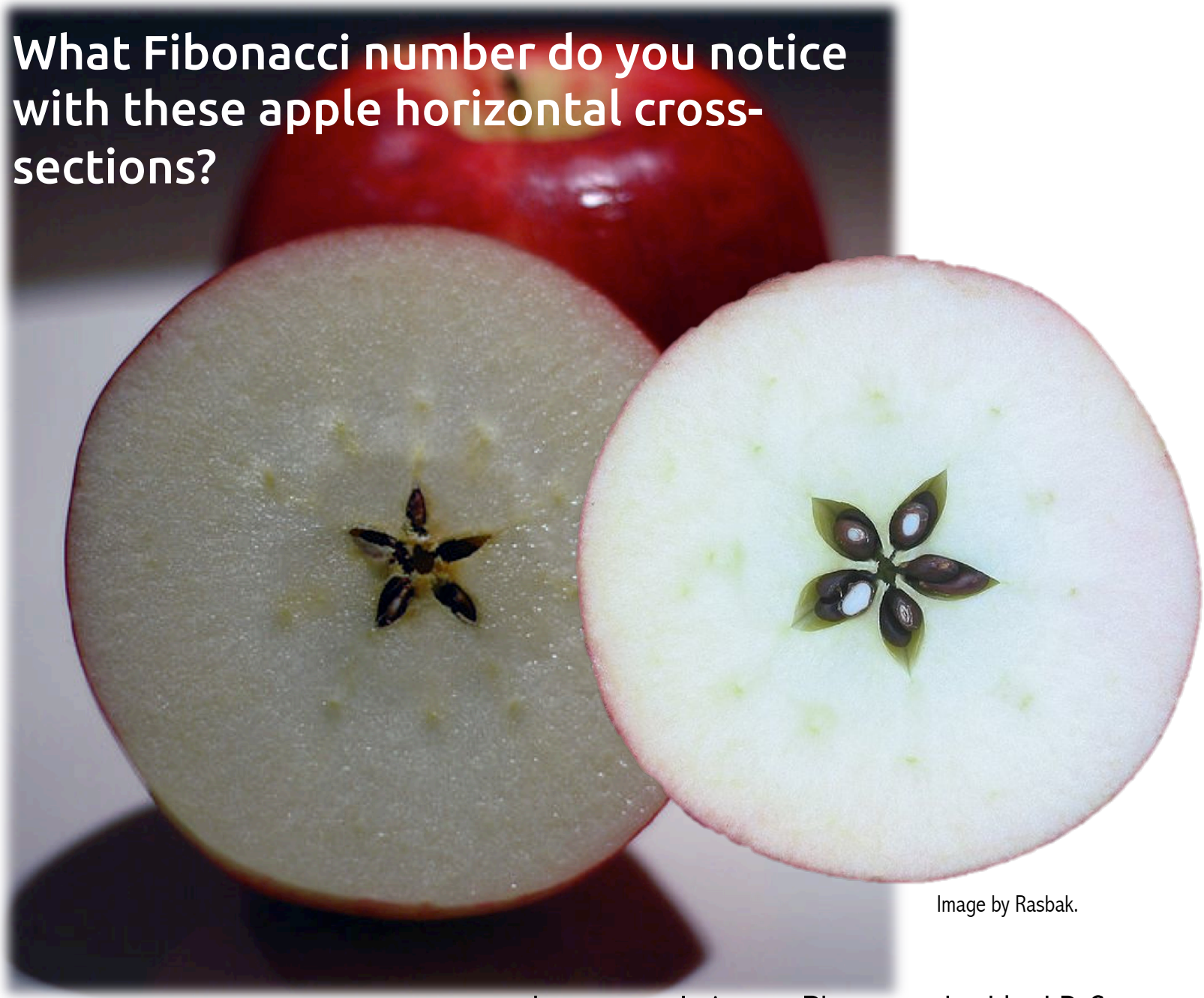


Image by Rasbak.

Larger apple image Photographed by LD Cross.

Fibonacci Sunflowers

- Seeds arranged in spirals
- One spiral set turns to left
- Another opposite spiral set turns right

- Counting spirals turns in each direction
- Almost always two numbers in Fibonacci series

Images – Background: A sunflower photographed by Amada; bottom and top frames, sunflowers from Pixabay (public domain).

References

1. Wikipedia Fibonacci number: https://en.wikipedia.org/wiki/Fibonacci number
2. Fibonacci Numbers and the Golden Section by Dr. Ron Knott: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fib.html
3. Smith College – Phyllotaxis: http://www.math.smith.edu/phyllo/About/index.html
4. Brousseau, A (1969), "Fibonacci Statistics in Conifers", Fibonacci Quarterly (7): 525–32

Background transparency - Willow oak leaves (an oak). Public domain.

An aloe plant photographed by Stan Shebs.

Sidebar image - Copper beech tree leaves photographed by David Hawgood.

Thank you for watching!

Vincent Van Gogh's Three Sunflowers 1888. Public domain.

Arnica Montana (background, thumbnail and sidebar) by C. A. M. Lindman (1856–1928). A public domain image.