



The Mathematics of Music



Music, Mathematics & Ancient History

- Musical sounds have numerical properties
- Ancient cultures - Chinese, Egyptian, Mesopotamians studied mathematical principles of sound
- Pythagoreans of ancient Greece - investigated musical scales in ratios of small integers
- Pythagoras quote - "all nature consists of harmony arising out of numbers"

Background Image - A spectrogram of a violin waveform, with linear frequency on the vertical axis and time on the horizontal axis. The bright lines show how the spectral components change over time. The intensity coloring is logarithmic (black is -120 dBFS). Photographed by Omegatron.

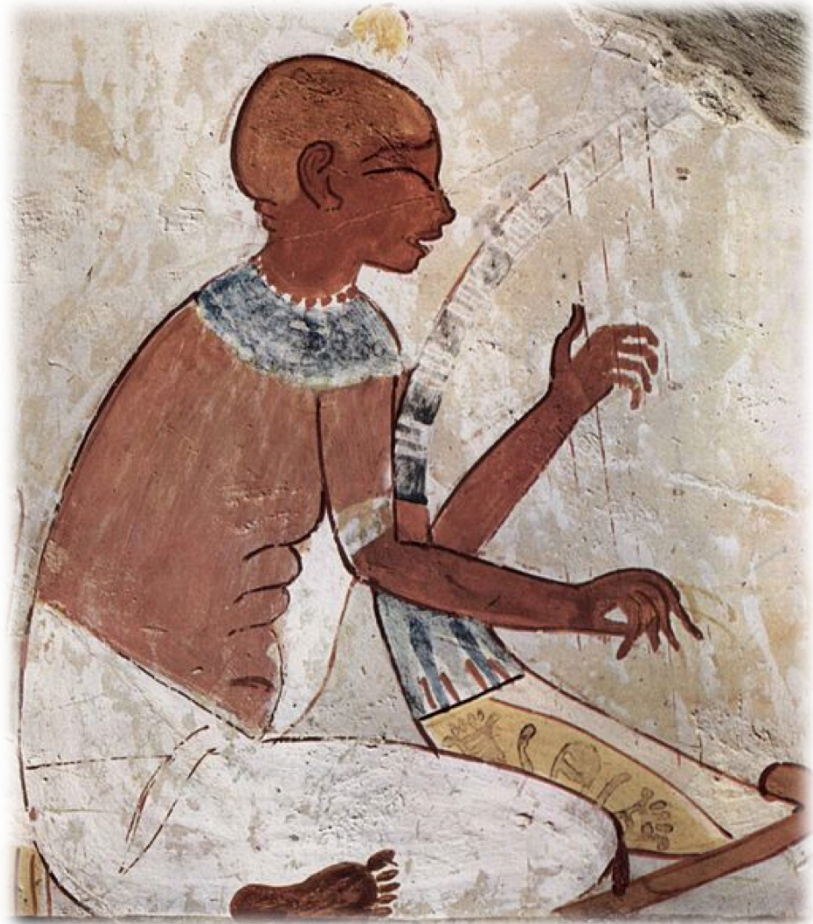


Terracotta figurine musicians playing the salpinx (trumpet) and the hydraulis (water organ). Photographed by Marie-Lan Nguyen.

Cover image – A violin
photographed by ZUU
Mumu Entertainment

Mathematical Laws of Harmonics & Rhythms

- **Ancients - sought to show mathematical laws of harmonics and rhythms**
- **Reading music – mathematics found as simple operations of counting and measuring**



**Image – an ancient Egyptian harpist.
Image in the public domain.**

Music Staff

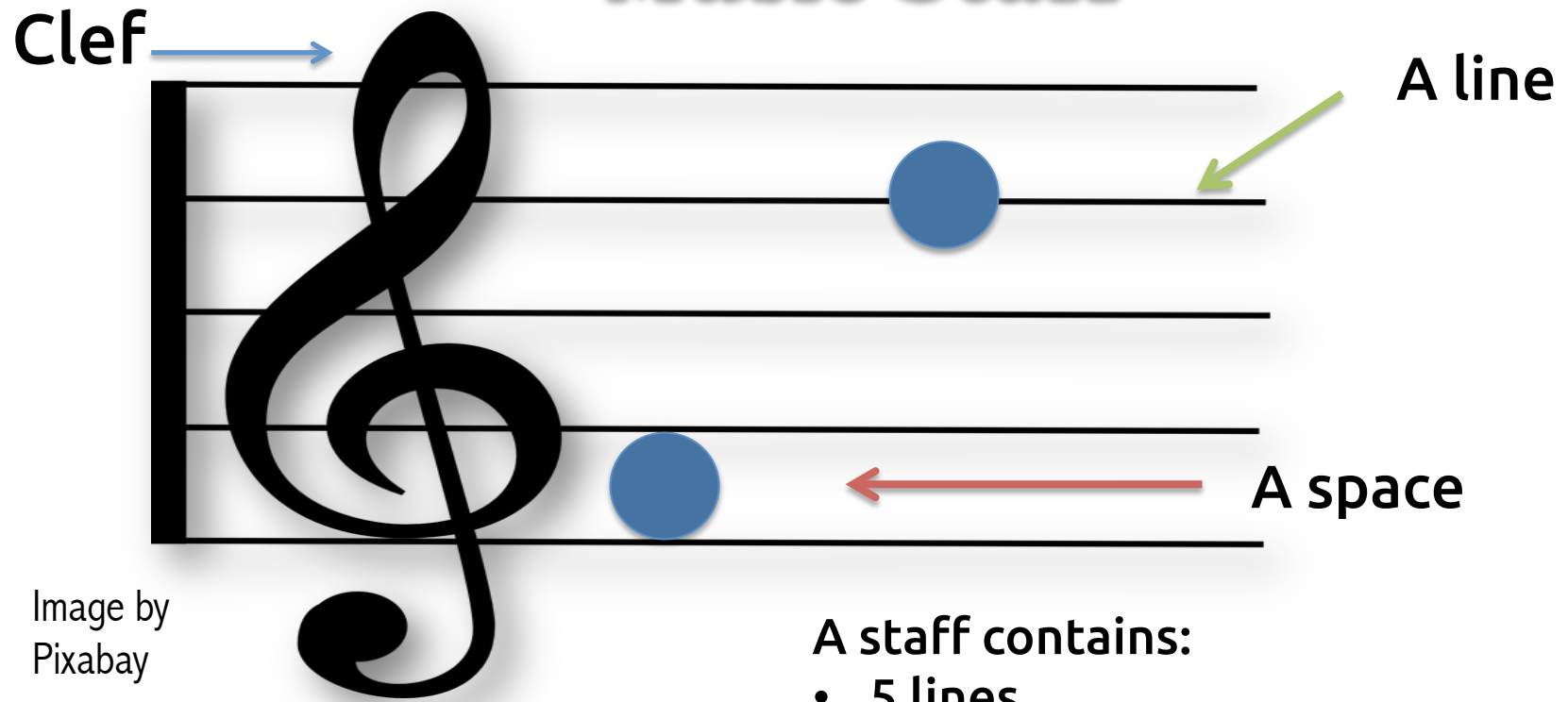


Image by
Pixabay

Each musical note has its place either on the lines or in the spaces.

A staff contains:

- 5 lines
- 4 spaces
- Each line & space represents a different musical pitch
- Percussion staff lines represent different instruments

Pitch & Tempo



- Pitch – the higher up the scale, the higher sound of note
- Tempo – note speed

Image by Classicalmusicfreak

Music Staff Bars



- Staff is divided into measures or bars with vertical lines; divides music according to meter
- Each bar represents a time period
- More notes within bar, more notes heard during time period



“Music is the pleasure the human mind experiences from counting without being aware that it is counting.”



**Gottfried Leibniz (1646 – 1716)
Co-discoverer of Calculus**



Music & Fractions

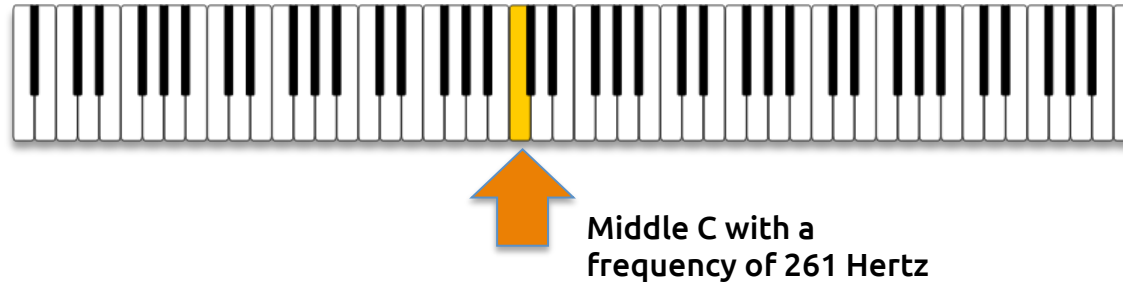


Note lengths

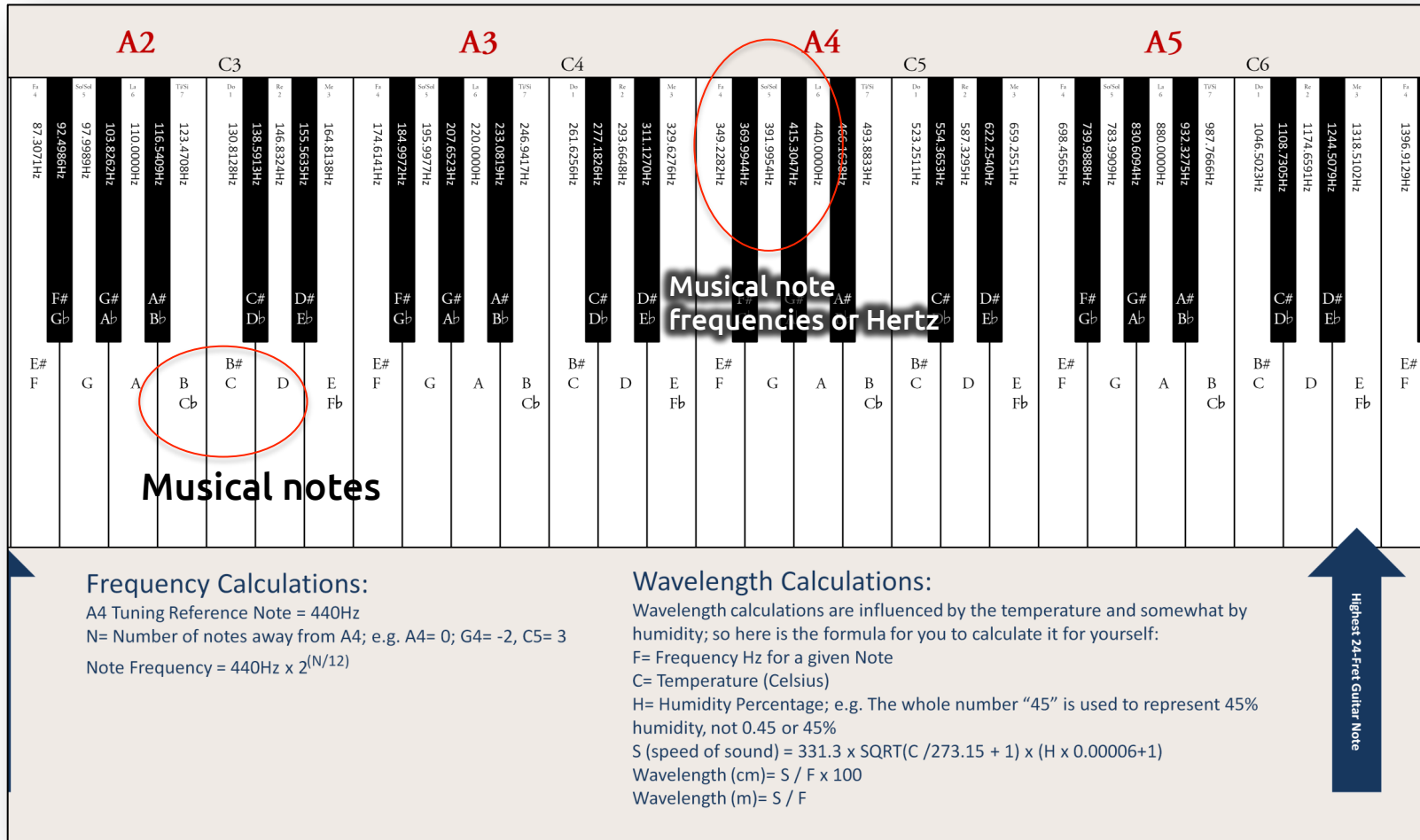
- Defined by how long last compared to whole note
- Are fractions
- Musicians are constantly doing fractions and counting when playing

Image by GNU Lilypond

Music & Sound Vibration



- Musical instruments vibrate when played
 - When plucked, string instruments vibrate
 - Higher pitch = More waves per second
-
- Vibration causes mechanical energy to travel through air in waves
 - Frequency – number of times per second sound waves hit our ear
 - Frequency measured in hertz
 - Hertz named after German, Heinrich Hertz who made important contributions to electromagnetism field



Musical Note Frequencies

A partial chart of all humanly audible musical note frequencies extending a little beyond the audible range. Includes interesting facts and the formulas to calculate the frequency and wavelength for any musical notes.

Image created by Markmain.

Musical Scale

- Musical scale - set of pitches used in making or describing music

Diatonic Scale

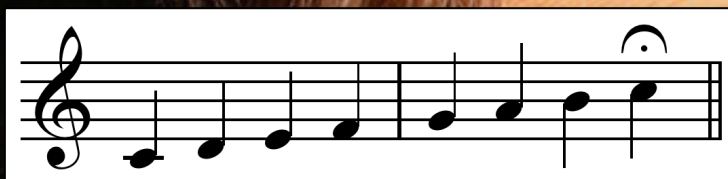


Image by Hyacinth

- Diatonic scale - most important scale in western music
- Each pitch - corresponds to particular frequency, expressed in hertz (Hz), sometimes referred to as cycles per second (c.p.s.)

Background - Young girl playing an acoustic guitar

The Golden Ratio and Fibonacci Numbers in Music

- Golden Ratio – mathematic ratio of beauty found throughout nature and used in art, architecture & music
- Fibonacci numbers - part of natural harmony of visual and sounding beauty; found in nature
- Fibonacci number sequence - number pattern 0,1,1,2,3,5,8,13 into infinity
- Music written in past and present use golden ratio and Fibonacci numbers to assist
- James Tenney - reconceived his piece "For Ann (Rising)" using golden ratio





Fibonacci Numbers & Music Cont'd



- Piano keyboard – link between music and Fibonacci Numbers (1, 1, 3, 5, 8, 13...)
- Keyboards octave – consists of 8 white keys and 5 black keys
- Black keys positioned in groups of 2 and 3
- 13 keys altogether in one octave
- All are Fibonacci numbers

- 13 notes belong to “chromatic scale”, most complete scale developed in Western music; previously 8-note diatonic scale, better known as octave which was preceded by 5-note *pentatonic scale*
- *Pentatonic scale* – used in early European music; basis today of American Kodaly method of music ed for younger children (Mary had a little lamb”, “Ring Around the Rosy”)

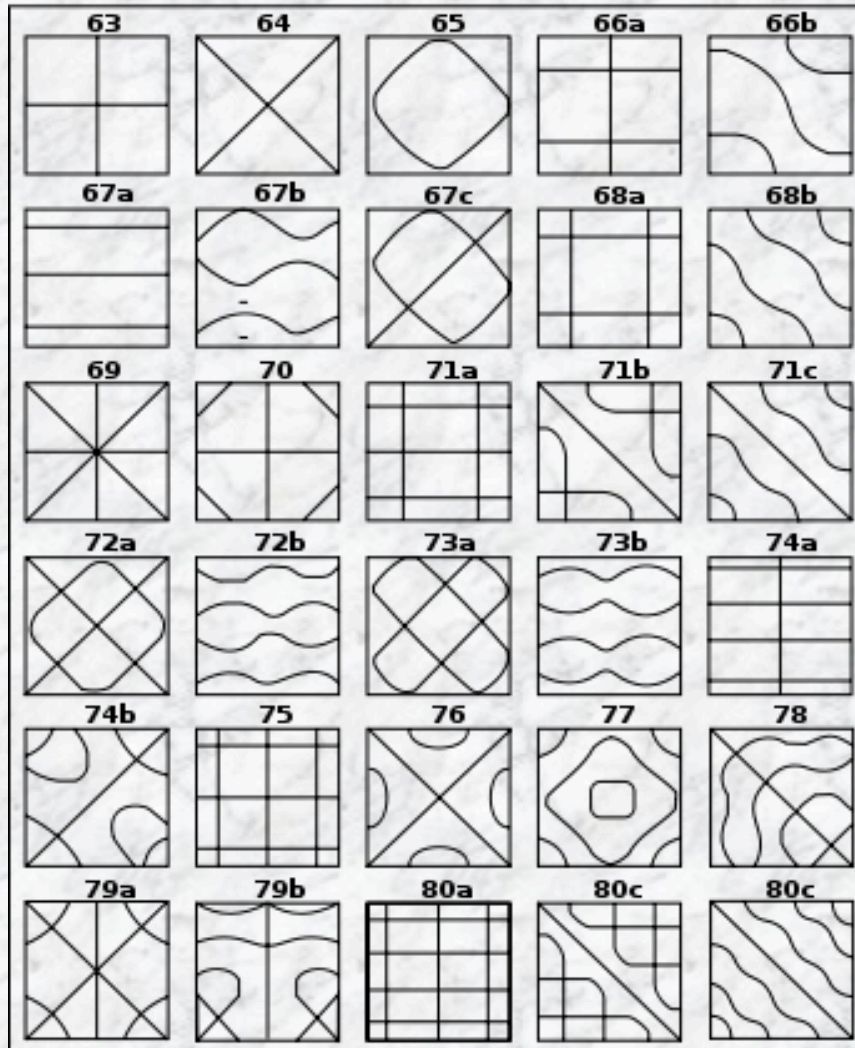
Pleasing Musical Intervals

- Musical intervals most pleasing – major sixth and minor sixth
- A major sixth – C, vibrating at about 264 vibrations per second; A, vibrating at about 440 vibrations per second
- Ratio of 264 to 440 reduces to $3/5$, a Fibonacci ratio



Background Image – an Hohner accordion

Chladni's Akustik



Chladni figure patterns produced by sound vibrations in fine powder on a square plate. (Ernst Chladni, Acoustics, 1802)

References Used

Background image & thumbnail - *Attributes of Music* by Anne Vallayer-Coster 1770 in the public domain.

1. Reginald Smith Brindle, *The New Music*, Oxford University Press, 1987, pp 42-3
2. Reginald Smith Brindle, *The New Music*, Oxford University Press, 1987, p 42
3. Plato, (Trans. Desmond Lee) *The Republic*, Harmondsworth Penguin 1974, page 340, note.
4. Harmony and Proportion - <http://www.aboutscotland.com/harmony/prop.html>
5. Wikipedia article, Music and mathematics: http://en.wikipedia.org/wiki/Music_and_mathematics

**Thank you
for
watching!**

